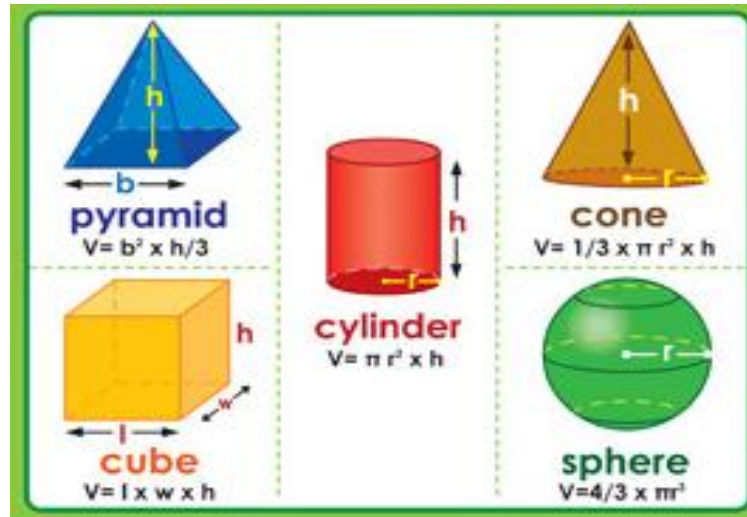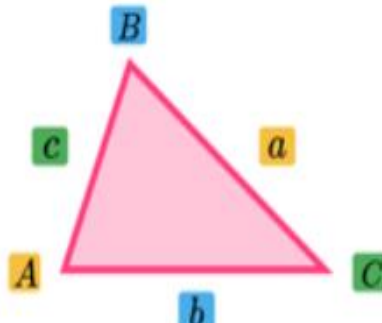# Military Technological College

Cosine Rule

This is the cosine rule to find a missing side:

$$a^2 = b^2 + c^2 - 2bc \cos(A)$$

This is the rearranged cosine rule to find a missing angle:

$$Cos(A) = \frac{b^2 + c^2 - a^2}{2bc}$$

Sine rule

This is the sine rule:

$$\frac{a}{sin(A)} = \frac{b}{sin(B)} = \frac{c}{sin(C)}$$

or

$$\frac{sin(A)}{a} = \frac{sin(B)}{b} = \frac{sin(C)}{c}$$

GFP- Pure Mathematics

MODULE CODE: MTCG1018

WORKBOOK- 1

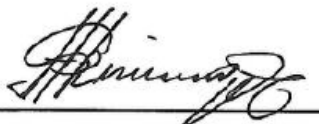| Learning Outcomes – On successful completion of this module, students should be able to:	
1.	Demonstrate understanding of the definition of a function and its graph.
2.	Define and manipulate exponential and logarithmic functions and solve problems arising from real life applications.
3.	Understand the inverse relationship between exponents and logarithms functions and use this relationship to solve related problems.
4.	Understand basic concepts of descriptive statistics, mean, median, mode and summarize data into tables and simple graphs (bar charts, histogram, and pie chart).
5.	Understand basic probability concepts and compute the probability of simple events using tree diagrams and formulas for permutations and combinations.
6.	Define and evaluate limit of a function as well as test continuity of a function.
7.	Determine the surface areas, the volumes and capacities of common shapes and 3-dimesions figures (square, rectangle, parallelogram, trapezium, cuboid, cone, pyramid and prisms).
8.	Find the derivatives of standard and composite functions using standard rules of differentiation.
9.	Use the law of sines and cosines to solve a triangle and real-life problems.

MILITARY TECHNOLOGICAL COLLEGE

Delivery Plan - Year 2024-25
[Term 2]

Title / Module Code / Programme	Pure Mathematics /MTCG1018/Foundation Programme Department (FPD)	Module Coordinator	Mr. Knowledge Simango
Lecturers	TBA	Resources & Reference books	Moodle & Workbook
Duration & Contact Hours	Term 2: 4 hrs x 11 weeks = 44 hours		

Week No.	TOPICS	Hours	Learning Outcome No.
1	**Introduction** **1. Law of sines and cosines to solve a triangle** 1.1 Law of sines 1.2 Law of cosines **2. Perimeter, Area and Volume** 2.1 Perimeter and area	4	7, 9
2	2.2 Volume and surface area **3. Statistics** 3.1 Basic concepts of descriptive statistics 3.2 Types of Data	4	4, 7
3	3.3 Summarizing and presenting data. 3.4 Measures of Central Tendency 3.5 Measures of Dispersion **Revision for Continuous Assessment-1**	4	4
	Continuous Assessment-1 (Chapter 1 and 2)		**7 and 9**
4	**4. Probability** 4.1 Basic Concepts 4.2 Probability 4.3 Rules of Probability	4	5
5	**5. Functions and graphs** 5.1 Domain, range and function 5.2 Types of functions 5.3 Inverse function	4	1
6	5.4 Operations of functions 5.5 Composite function **6. Exponential functions** 6.1 Exponential equations	4	1

7	6.2 Exponential function and graphs 6.3 Application in real life **Revision for Continuous Assessment-2**	4	2
	Continuous Assessment-2 (Chapter 3, 4 and 5)		1, 4 and 5
8	**7. logarithmic functions** 7.1 Logarithm Definition and Properties 7.2 Logarithmic function and graph 7.3 Exponential and logarithmic equations **8. Limits** 8.1 Basic Concepts of Limit	4	2, 3, 6
9	8.2 Methods of finding limits 8.3 Limits at Infinity 8.4 Continuity of a Function **9. Differentiation** 9.1 The Gradient of a Curve	4	6, 8
10	9.2 Differentiation from the First Principles 9.3 Methods of Differentiation	4	8
11	9.4 Applications of Derivatives	4	8
	Revision for Final Exam,		2, 3, 6 & 8
12/13	**FINAL EXAM (Unit-6 to Unit-9)**		2, 3, 6 & 8
	Total hours	44	

Indicative Reading	
Title/Edition/Author	**ISBN**
College Algebra with Trigonometry-7th Edition by K Raymond A., Ziegler Michael R., Byleen	ISBN-13: 978-0072368697 ISBN-10: 0072368691
College Algebra and Trigonometry-5th Edition by Margaret L. Lial, John Hornsby, David I. Schneider and Callie Daniels	ISBN-13: 978-0321671783 ISBN-10: 0321671783
Bird's Basic Engineering Mathematics- 8th Edition by John Bird	ISBN-13: 978-0367643706 ISBN-10: 0367643707
Engineering Mathematics- 8th Edition by K.A. Stroud and Dexter Booth	ISBN-13: 978-1352010275 ISBN-10: 1352010275

Mr. Knowledge Simango

Module Coordinator

Dr. T Raja Rani

DHOD FPD(CMP)

MQM Salim Al Shibli

Head FPD

Contents

Assessment Plan (Passing Mark: 50 %)

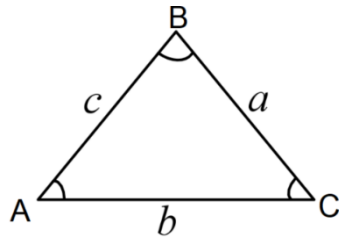| Assessment	Weightage
Continuous Assessment-1	20%
Continuous Assessment-2	30%
Final Exam	50%
Total	100%

Note: Only Non-Programmable calculators, prescribed in MTC exam rules, are allowed.
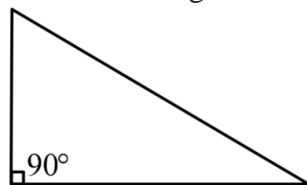
Attendance Policy:

Warning	Absence
First	10%
Second	15%
Third	20%

(UNIT-1) LAW OF SINES AND COSINES

In a triangle, there are always three vertices, three angles and three sides.

A **right (right-angled) triangle** is a triangle in which one angle is 90^0.

Right triangle

An **oblique triangle** is a **triangle** with no right angle. An **oblique triangle** has either three acute angles, or one obtuse angle and two acute angles.
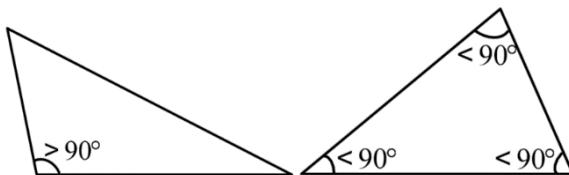
Obtuse triangle	Acute triangle

Oblique triangles

In any **triangle**, the sum of all three angles is equal to 180 degrees.

Notations used in solving triangle
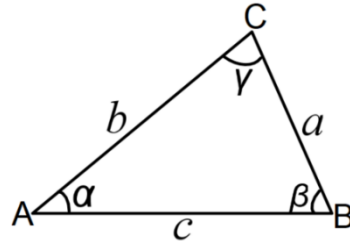
For example in the following triangle:

$a = 500$; $b = 940$; $c = 985$

$\alpha = 30°$; $\beta = 70°$; $\gamma = 80°$
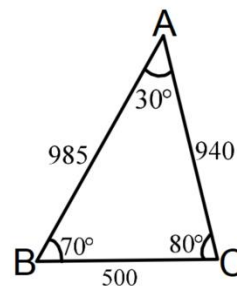
Note: Authors also use A, B, and C for α, β and γ respectively.

Class Activity 1

Identify a, b, c, α, β and γ in the following triangle.

Answer:

a =   ;   b =   ;   c =

$\alpha =$; $\beta =$; $\gamma =$

1
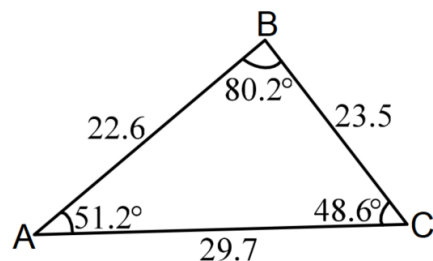
The law of sines and law of cosines play an important role in solving oblique triangles.

1.1 LAW OF SINES
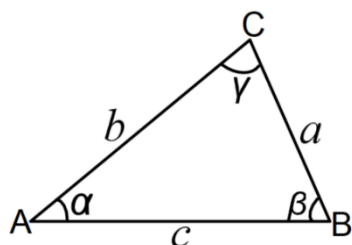
Any side of a triangle is proportional to the sine function of its opposite angle. As per the sine law:

$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}$$

Or $\quad \dfrac{a}{\sin\alpha} = \dfrac{b}{\sin\beta} = \dfrac{c}{\sin\gamma}$
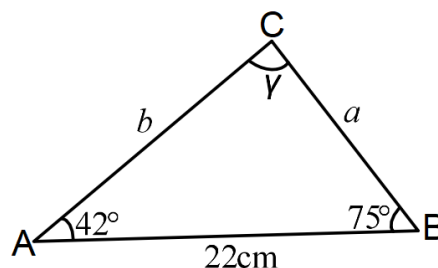
The law of sines is used to solve triangles in the following cases:

(i) Two angles and any side (ASA or AAS)

(ii) Two sides and an angle opposite one of them (SSA)

Example 1: Solve the triangle:

(Round the answers up to 2 decimal places)

Solution: We are given two angles and the included side, which is the ASA case. Here

$\alpha = 42°, \beta = 75° \ and \ c = 22 \, cm$

Step (1) Find the third angle

$\alpha + \beta + \gamma = 180°$

$42° + 75° + \gamma = 180°$

$\gamma = 63°$

Step (2) Find any of the remaining two sides with the Sine rule

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$

$$\frac{a}{sin\,4\,2°} = \frac{b}{sin\,7\,5°} = \frac{22}{sin63°} \quad ......(1)$$

From first and third fraction of (1)

$$\frac{a}{sin\,4\,2°} = \frac{22}{sin63°}$$

$$a = \frac{22\,sin\,4\,2°}{sin\,6\,3°} = 16.52 \ cm$$

(Rounded to 2 decimal places)

Step (3) Find the third side with the Sine rule
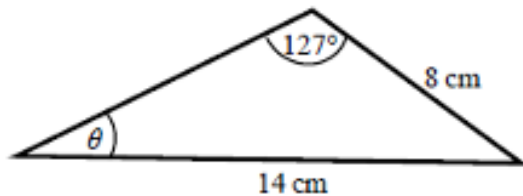
$$\frac{b}{sin\,75°} = \frac{22}{sin63°}$$

From second and third fraction of (1)

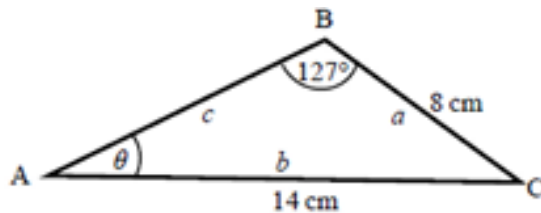$$b = \frac{22\,sin\,7\,5°}{sin\,6\,3°} = 23.85 \ cm$$

(Rounded to 2 decimal places)

Example 2: Find the value of angle θ

(Round off the answers to 1 decimal place)

Step 1: Start by labelling the angles and sides of the triangle.

Step 2: Write the sine rule to find the angle.

$$\frac{sinA}{a} = \frac{sinB}{b}$$

Step 3: Substitute the values of a, b, A and B into the formula.

$$\frac{sin\theta}{8} = \frac{sin127°}{14}$$

Step 4: Rearrange to make $sin\theta$ the subject

$$sin\theta = \frac{8 \times sin127°}{14}$$

Step 5: Use sin^{-1} to find angle and round your answer to 1 decimal place.

$$\theta = sin^{-1}\left(\frac{8 \times sin127°}{14}\right)$$

$$\theta = 27.15°$$

$$\theta = 27.2° \ (\boldsymbol{1 \ decimal \ place})$$

1) Solve the triangle:

2) Solve the triangle:
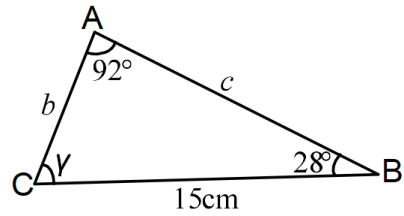
3) Calculate the angle θ
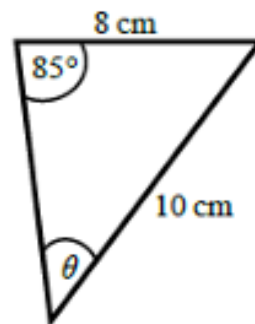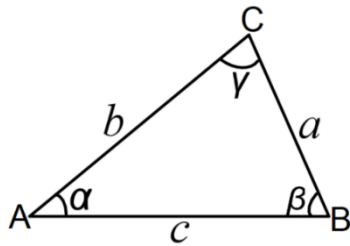
1.2 LAW OF COSINES

1) $a^2 = b^2 + c^2 - 2bc\,\cos\alpha$

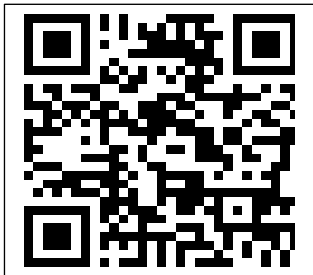2) $b^2 = a^2 + c^2 - 2ac\,\cos\beta$

3) $c^2 = a^2 + b^2 - 2ab\,\cos\gamma$

From 1) $\alpha = \cos^{-1}\left(\dfrac{b^2+c^2-a^2}{2bc}\right)$

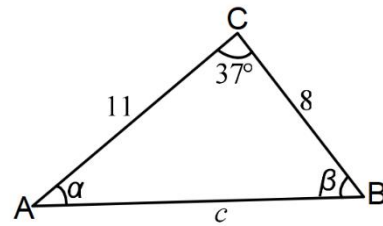From 2) $\beta = \cos^{-1}\left(\dfrac{a^2+c^2-b^2}{2ac}\right)$

From 3) $\gamma = \cos^{-1}\left(\dfrac{a^2+b^2-c^2}{2ab}\right)$

Note: The law of cosines is used to solve triangles in the following cases:

(i) Two sides and an included angle (SAS)

(ii) Three sides (SSS)

Example 1: Solve the following triangle.

(Round the answers up to 2 decimal places)

Solution: Here

$\gamma = 37°, a = 8,\ and\ b = 11$

We are given two sides and an included angle, which is the SAS case.

Step (1) Find the third side with the Cosine rule

$c^2 = a^2 + b^2 - 2ab\,\cos\gamma$

$c^2 = 8^2 + 11^2 - 2(8)(11)\,\cos 37$

$c = 6.6663...$

Step (2) Find any one of the remaining two angles with the Cosine rule

For finding β the rule is

$$\beta = \cos^{-1}\left(\frac{a^2 + c^2 - b^2}{2ac}\right)$$

$$\beta = \cos^{-1}\left(\frac{8^2 + 6.6663^2 - 11^2}{2 \times 8 \times 6.6663}\right)$$

$$\beta = 96.7628\ ...$$

$$\beta = 96.76°$$

(Rounded to 2 decimal places)

Step (3) Find the third angle

Method 1) $\alpha + \beta + \gamma = 180°$

$\alpha + 96.76° + 37° = 180°$

$\alpha = 46.24°$

Method 2) $\alpha = cos^{-1}\left(\frac{b^2+c^2-a^2}{2bc}\right)$

$\alpha = cos^{-1}\left(\frac{11^2 + 6.6663^2 - 8^2}{2 \times 11 \times 6.6663}\right)$

$\alpha = 46.2371 \dots = 46.24°$

(Rounded to 2 decimal places)

$\therefore \alpha = 46.24°, \quad \beta = 96.76°$

$and\ c = 6.67$ (Rounded to 2 decimal places)

Alternative method after step 1

The angle can also be found with the help of sine rule but first will have to find the angle opposite the shorter of the two given sides. This angle will always be acute as in a triangle as there cannot be two obtuse angles in a triangle.

$\frac{sin\ \alpha}{8} = \frac{sin\ \beta}{11} = \frac{sin37°}{6.6663}$
---(1)

Here we will have to find first α

$$\frac{sin\ \alpha}{8} = \frac{sin\ 37°}{6.6663}$$

or

$$sin\ \alpha = \frac{8\ sin\ 37°}{6.6663}$$

Hence,

$$\alpha = sin^{-1}\left(\frac{8\ sin\ 37°}{6.6663}\right)$$
$$\alpha = 46.2378\dots° = 46.24°$$
(Rounded to 2 decimal places)

Step (3) Find the third angle

$\alpha + \beta + \gamma = 180°$

$\qquad 46.24° + \beta + 37° = 180°$

$\qquad \beta = 96.76°$

$\therefore \alpha = 46.24°, \beta = 96.76°$

$and\ c = 6.67$ (Rounded to 2 decimal places)

Note: If the third angle is found by Sine rule in this question then it will give wrong answer as it is an obtuse triangle.
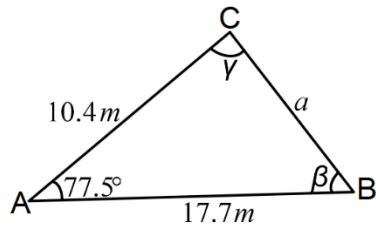
Class Activity 3

(Round the answers up to 2 decimal places)
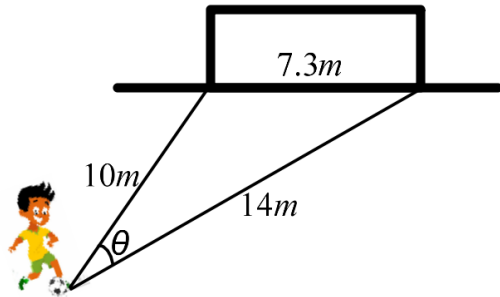
1) Solve the following triangle:

6
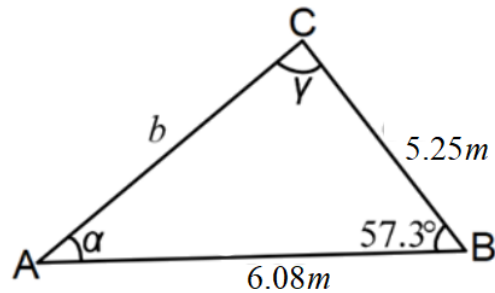
2) Solve the following triangle:

3) Find the missing side and angles of the triangle with: $\alpha = 30°$, $b = 12$, $c = 24$.

4) A soccer player takes a shot on a standard net that is 7.3 *m* wide. If the player is 10 *m* from one goalpost and 14 *m* from the other, through what angle θ can a goal be made?

WORKSHEET 1

Section-A

Circle the correct answer in the following questions.

1) Sum of all three angles of a triangle is equal to …………..

(a) $270°$

(b) $90°$

(c) $180°$

2) An oblique triangle has …………..

(a) one obtuse angle

(b) a right angle

(c) two obtuse angles

Section-B

Show your solution step by step in the following questions. Round off the answer to two decimal places.

1) In the following triangle calculate side 'AB'.
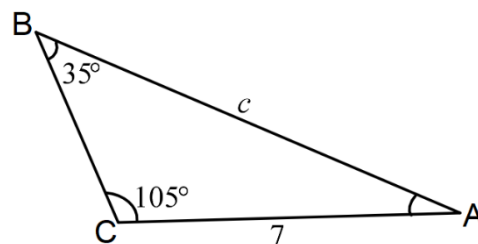
Solution:

2) Solve the triangle:
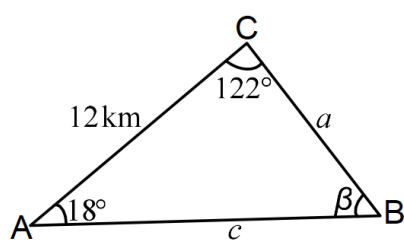
Solution:

3) Find the missing side and angles of the triangle with: $\beta = 26°$, $a = 26$, $c = 18$.

Solution:

4) A satellite calculates the distances and angle shown in (Figure) (not to scale). Find the distance between the two cities. (***Round answers to the nearest tenth***).
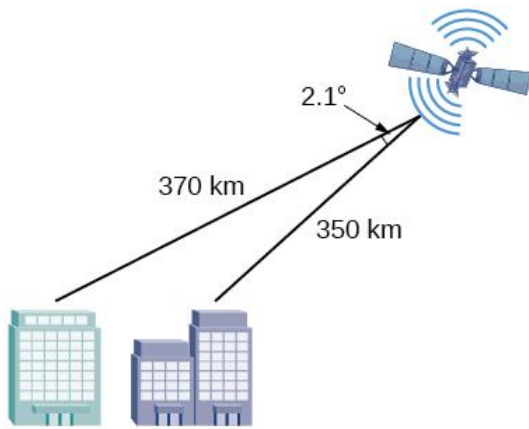
Solution:

5) A man leaves a point 'A' walking a distance of $32.5 km$ in a direction of $70°$. A cyclist leaves the same point at the same time in a direction $130°$ travelling at a constant speed. If the walker and the cyclist are 80 *km* apart, find the distance travelled by the cyclist.

Solution:

11
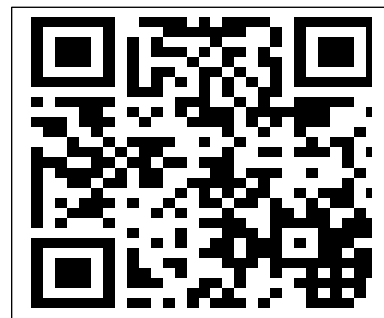
(UNIT-2) PERIMETER, AREA AND VOLUME
2.1 PERIMETER AND AREA

Perimeter

The perimeter of a shape is the distance all the way round its edges.

Perimeter is measured in units such as centimetres, feet, metres, etc.

Example 1: The perimeter of the following figure is

$4\ m + 5\ m + 4\ m + 5\ m = 18\ m$
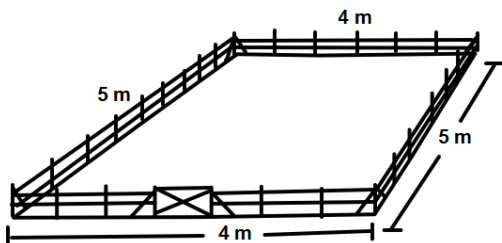
Area

The area of a surface is the amount of square length units contained in the surface.

Note: 1 *sq. m* means that 1*m squares* with 1*m* on each side, can be placed precisely on a surface.

Example 1: Consider a room 4 *m* by 3 *m* as shown below.
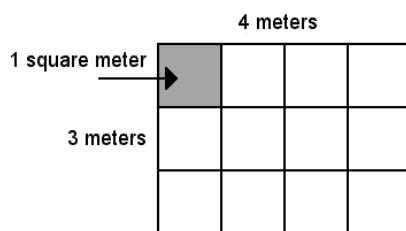
Clearly it can be divided up into 12 equal squares, each measuring 1 *m* by 1 *m*. Each square has an area of 1 *square meter*.
Hence, the total area is 12 *m²*. So, to calculate the area of a rectangle, multiply length of one side by the length of the other side.

Note: $Area = 4\ m \times 3\ m = 12\ m^2$

The perimeter and area of some polygons is given below:
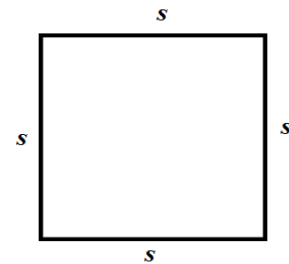
Triangle:
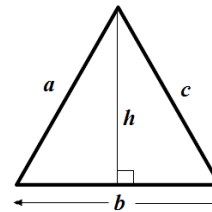
$P = a + b + c$

$A = \dfrac{1}{2}bh$

Rectangle:
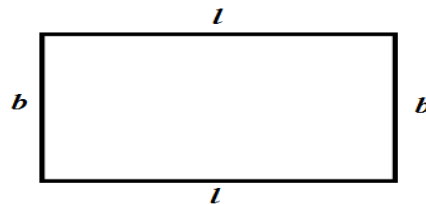
$P = 2l + 2b$

$A = lb$

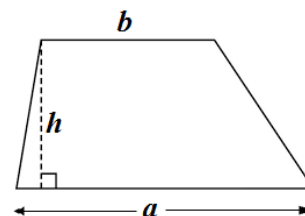**Square**:

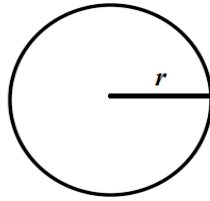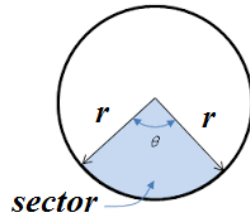

$P = 4s$

$A = s^2$

Trapezium:

$A = \dfrac{1}{2}(a+b)h$

Circle:

$Circumference(C) = 2\pi r = \pi d$

$A = \pi r^2$

Sector:
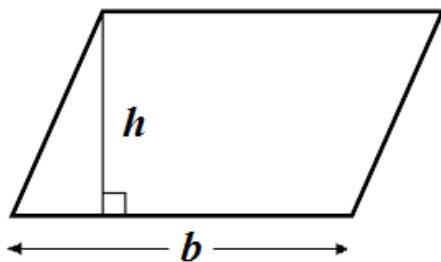

sector

Length of the arc:

$L = \theta r$ if θ is in radians

$L = \theta \left(\dfrac{\pi}{180°}\right) r$ if θ is in degrees

Area of Sector:

$$A = \frac{1}{2} Lr$$

$A = \dfrac{\theta}{2} r^2$ if θ is in radians

$A = \theta \left(\dfrac{\pi}{360°}\right) r^2$ if θ is in degrees

Parallelogram:

$A = b h$

Perimeter

Area
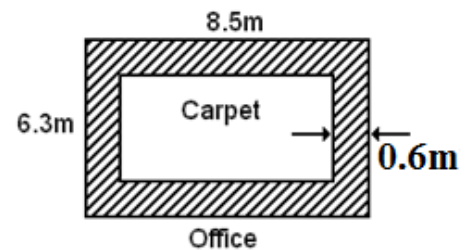
Examples:

1) Find the area in the following figure.
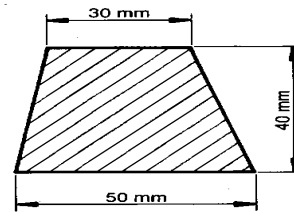
Solution: Area of trapezium

$$= \frac{1}{2}(40)(30+50)$$

$$= 1600 \ mm^2$$

2) An office 8.5 m by 6.3 m is to be fitted with a carpet, so as to leave surround 0.6 m wide around the carpet. What is the area of the shaded region?
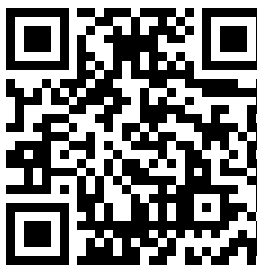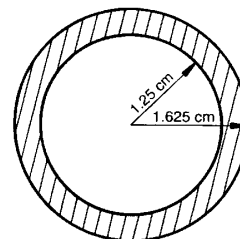
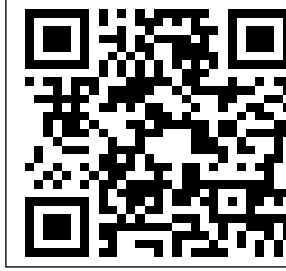**Solution:**

$Office\ area = (8.5)(6.3) = 53.55 m^2$

$Carpet\ area = (8.5 - 2 \times 0.6)(6.3 - 2 \times 0.6) = 37.23 m^2$

$The\ area\ of\ shaded\ region = 53.55 - 37.23 = 16.32 m^2$

3) Calculate the cross-sectional area of the pipe.

Area of cross-section

$= $ area of outside circle - area of inside circle

$= \pi(1.625)^2 - \pi(1.25)^2$

$= 3.39 cm^2$

4) Calculate the length of arc of a circle whose radius is 8 *m* and which subtends an angle of 56° at the centre, and the area of the sector so formed.

Solution:

$$L = \theta \left(\frac{\pi}{180°}\right) r = 56 \left(\frac{\pi}{180°}\right) 8$$
$$= 7.82m$$

$$A = \theta \left(\frac{\pi}{360°}\right) r^2 = 56 \left(\frac{\pi}{360°}\right) (64)$$
$$= 31.28m^2$$

Class Activity

1) Find the area of a triangle whose base is 8 *cm* and altitude 4.5 *cm*.
Solution:

2) The area of a rectangle is 220 *mm²*. If its length is 25 *mm*, find its width.
Solution:

3) If the perimeter of a square is 40 *m*, find its area.
Solution:

4) Find the area of the trapezium whose parallel sides are 75 *mm* and 82 *mm* long respectively and whose vertical height is 39 *mm*.
Solution:

5) Find the circumference and area of a circle whose diameter is 1.6 *m*.
Solution:

6) Find the diameter of a circle whose circumference is 34.4 *cm*.
Solution:

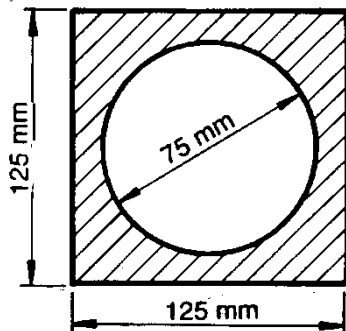7) Find the area of a parallelogram whose base is 120 *cm* and height 11 *cm*.
Solution:

8) The following diagram shows a sector of a circle, centre O. The radius of the circle is 12 *cm*. Angle AOB = 60°. Find the perimeter of the sector.
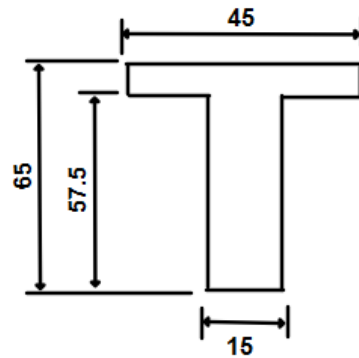

60°

A

B

O
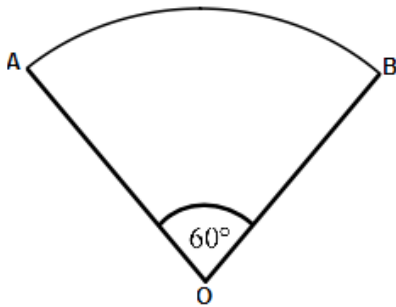
Solution:

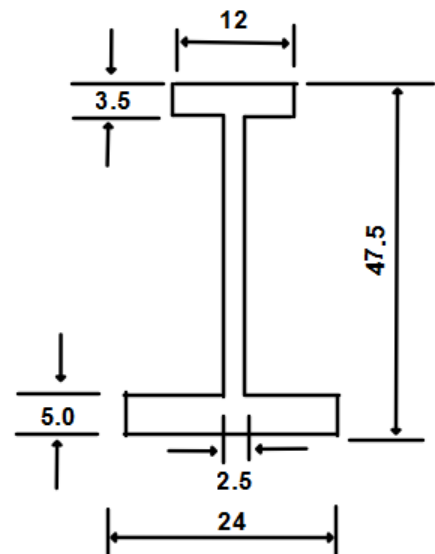9) Find the area of the shaded portion in the following figure:

125 mm

75 mm

125 mm

Solution:

10) Find the area of the section in the following figure:

45

65

57.5

15

Solution:

11) Find the area of the section in the following figure:
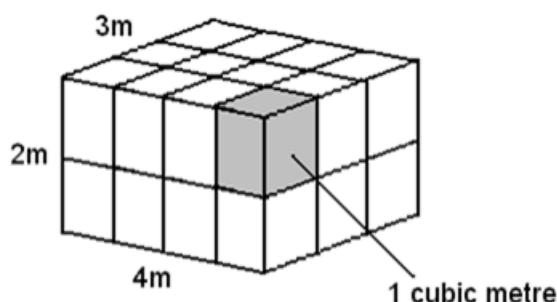

12

3.5

47.5

5.0

2.5

24

Solution:

2.2 VOLUME AND SURFACE AREA

Volume

The concept and calculation of volume is in the logical extension of length and area.

Instead of squares, we now consider cubes. This is a 3-dimensional concept and the typical units of volume are **cubic metres** (m^3).

If we have a box, length 4 *m*, width 3 *m* and height 2 *m*, we see that the total volume = 24 *cubic metres* (24 m^3).

Each layer = (4)(3) = 12 cubes.
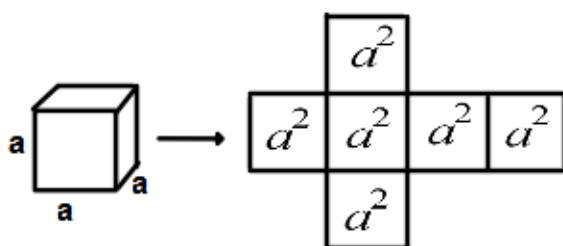
There are 2 layers.

Hence the volume = (12)(2) = 24 m^3.

Basically, when calculating volume, it is necessary to look for 3 dimensions, at 90° to each other, and then multiply them together. For a box shape,

Volume = (length)(width)(height)

Total Surface Area (TSA)
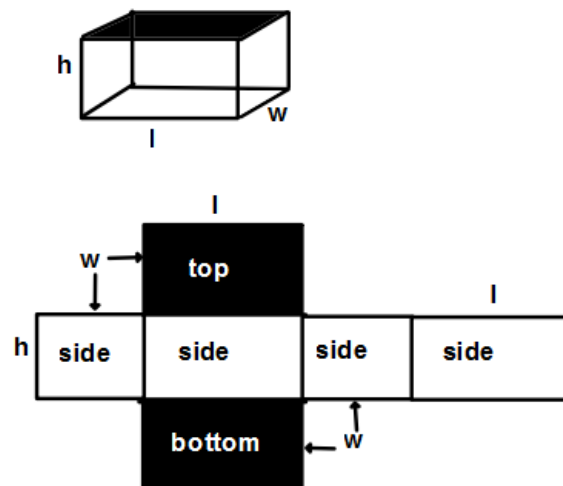The total surface area is the sum of all the areas of all the shapes that cover the surface of the object.

$$\text{Surface area of a Cube} = 6a^2$$

Lateral Surface Area (LSA)

The lateral surface of an object is the area of

all the sides of object excluding area of its base and top.

Note: When the face is a circular region or a smooth closed curve (without edges), the lateral surface is called **curved surface** (as it looks smooth and curved). Thus cylinder, cone etc. have curved surfaces. But cuboid, cube, triangular pyramid, pentagonal prism, etc. have lateral surfaces.
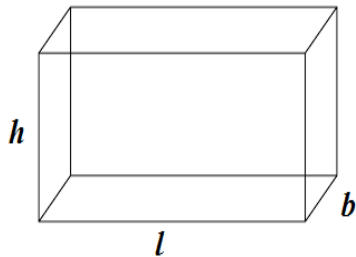

Lateral Surfaces

Circular Cone
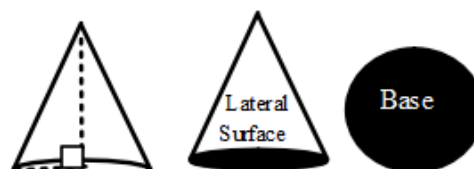
The volume, lateral or curved surface area and total surface area of the standard figures is given below:
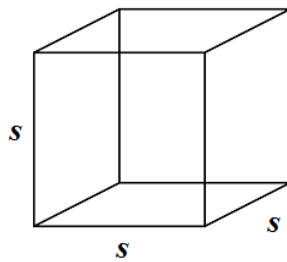
Cuboid:

$$V = l\,b\,h$$

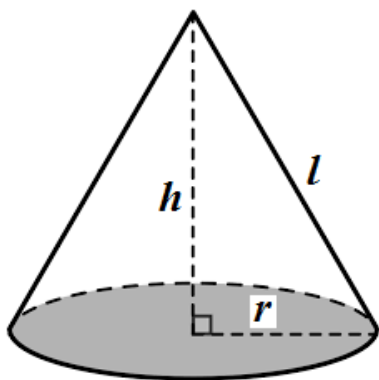$$LSA = 2\,h\,(l+b)$$

$$TSA = 2(l\,b + b\,h + h\,l)$$

Cube:

$$V = S^3$$

$$LSA = 4\,s^2$$

$$TSA = 6\,s^2$$

Cone:

$$V = \frac{1}{3}\pi\,r^2 h$$

$$CSA = \pi\,r\,l$$

$$TSA = \pi\,r^2 + \pi\,r\,l$$

Cylinder:
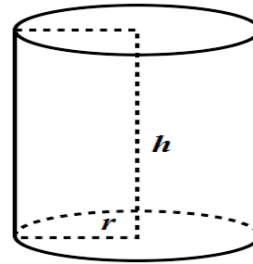
$$V = \pi\,r^2 h$$

$$CSA = 2\,\pi\,r\,h$$

$$TSA = 2\pi\,r(r+h)$$

Sphere:
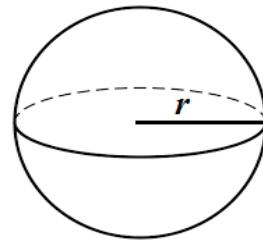
$$V = \frac{4}{3}\pi\,r^3$$

$$CSA = TSA = 4\pi\,r^2$$

Pyramid:

Tringular Base   Rectangular Base   Square Base
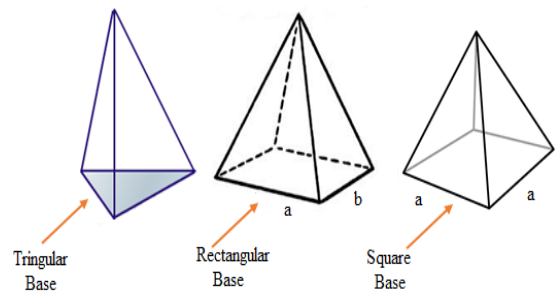
$$V = \frac{1}{3}\,Area\ of\ the\ base \times Height$$

$$= \frac{1}{3}\,A\,h$$

Prism:
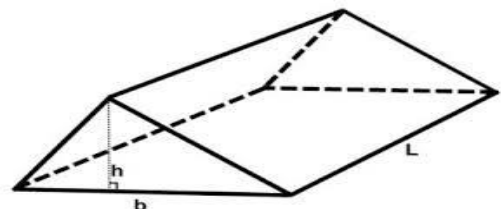
$$V = Area\ of\ the\,cross\ section \times Height$$

$$= \left(\frac{1}{2}bh\right)L$$

Class Activity

1) Find the lateral surface area, total surface area and volume of a box whose length, breadth and height are 20 *cm*, 15 *cm*, and 25 *cm* respectively.
Solution:

2) Find the lateral surface area of a cube of side 5 *cm*.
Solution:

3) If the volume of a cube is $8\ cm^3$, find its total surface area.
Solution:

4) Find the curved surface area of a cone whose height is 10 *cm* and diameter 8 *cm*.
Solution:

5) Find the curved surface area and total surface area of a cylinder of diameter 10 *cm* and height 10 *cm*
.**Solution:**

6) Find the volume and surface area of a sphere whose radius is 2 *cm*.
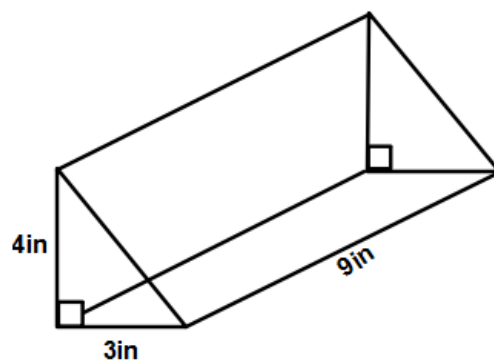**Solution:**

7) Calculate the diameter of a cylinder whose height is same as its diameter and whose volume is $220\ cm^3$.
Solution:

8) A pyramid has a square base of side 4 *cm* and a height of 9 *cm*. Find its volume.
Solution:

9) What is the volume of the following triangular prism?

4in

9in

3in

Solution:

WORKSHEET 2

Section-A

Circle the correct answer in the following questions.

1) If the perimeter of a square is 60 m then its side is …………..

(a) 20 m

(b) 15 m

(c) 30 m

2) If the perimeter of a square is 12 m, then its area is…………..

(a) $48m^2$

(b) $36m^2$

(c) $9m^2$

3) The perimeter of a parallelogram whose sides are 10 cm and 8 cm is ……

(a) 18 cm

(b) 36 cm

(c) 80 cm

4) The length of the arc JK in the following figure is …………..
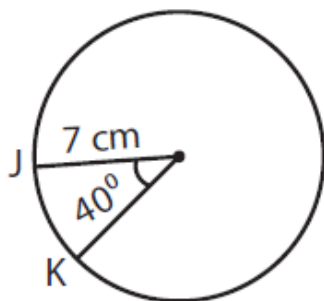
(a) 4.89 cm

(b) 280 cm

(c) 11.46 cm

Section-B

Show your solution step by step in the following questions. Round off the answer to two decimal places.

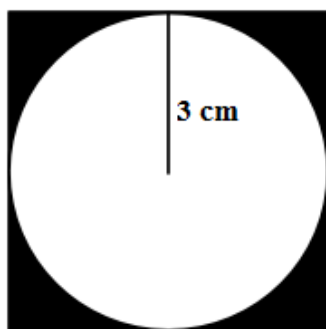1) If the area of a square is 25 m^2, find its perimeter.

Solution:

2) If the area of a circle is 50 cm^2, find its diameter.

Solution:

3) The radius of a sector is 3 *cm* and corresponding arc length is 6π *cm*. find the area of the sector.
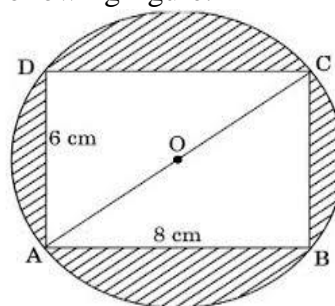
Solution:

4) Find the area of the shaded portion in the following figure:

3 cm

Solution:

5) Find the area of the shaded portion in the following figure:
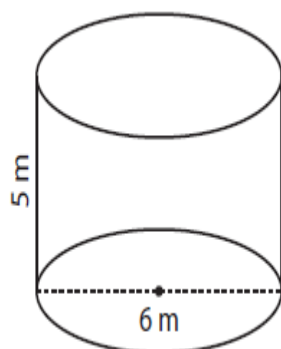

Solution:

6) If the total surface area of a cube is $54\ cm^2$, find its volume.

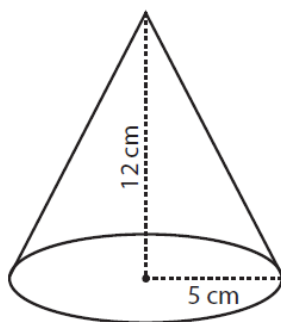**Solution:**

7) Find the volume of a cylinder whose height is 5 *m* and diameter 6 *m*.

6 m

Solution:

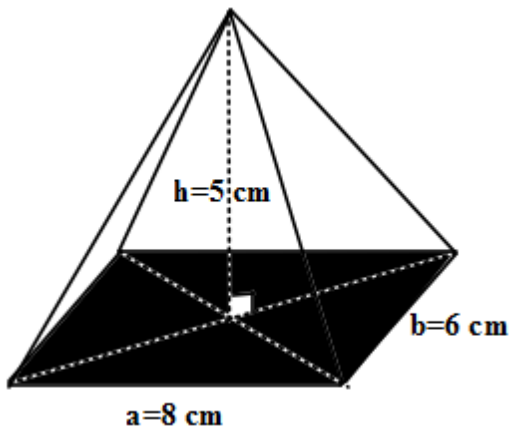8) Find the total surface area of the following figure.

5 cm

Solution:

9) Calculate the radius of a cylinder if its height is twice as its diameter and volume is $256\pi \ cm^3$.

Solution:

10) The surface area of a sphere is $36\pi \ cm^2$. Find its diameter.

Solution:
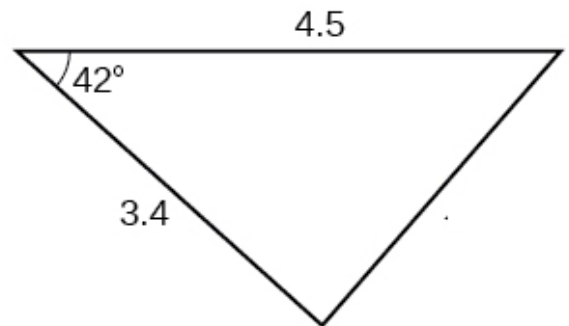
11) Find the volume of a rectangular-based pyramid whose base is 8 *cm* by 6 *cm* and height is 5 *cm*.

h=5 cm

b=6 cm

a=8 cm

Solution:

12) Find the perimeter of the following triangle:

4.5

42°

3.4

Solution:

REFERENCES AND INDICATIVE READING

Title/Edition/Author	ISBN
1. **College Algebra with Trigonometry-7th Edition** K Raymond A., Ziegler Michael R., Byleen K.	ISBN-13: 978-0072368697 ISBN-10: 0072368691
2. **College Algebra and Trigonometry-5th Edition** Margaret L. Lial, John Hornsby, David I. Schneider &Callie Daniels	ISBN-13: 978-0321671783 ISBN-10: 0321671783
3. **Bird's Basic Engineering Mathematics- 8th Edition** John Bird	ISBN-13: 978-0367643706 ISBN-10: 0367643707
4. **Engineering Mathematics- 8th Edition** K.A. Stroud and Dexter Booth	ISBN-13: 978-1352010275 ISBN-10: 1352010275
5. **Introduction to Statistics-3rd Edition** Ronald E. Walpole	ISBN-13 : 978-0024241405 ISBN-10 : 0024241407

Websites:

i) http://www.statisticshowto.com

ii) http://math.tutorvista.com

iii) www.mathsisfun.com

iv) www.statcan.gc.ca

v) http://www.analyzemath.com

vi) https://www.hippocampus.org