Military Technological College

BASIC MATHEMATICS

WORKBOOK-3 **MODULE CODE: MTCG1016**

MILITARY TECHNOLOGICAL COLLEGE

Delivery Plan - Year 2024-25 [Term 1]

Title / Module Code / Programme	Basic Mathematics /MTCG1016/Foundation Programme Department (FPD)	Module Coordinator	Rajendar Palli
Lecturers	TBA	Resources & Reference books	Moodle & Workbook
Duration & Contact Hours	Term 1: 5 hrs x 11 weeks = 55 hours		

WEEK No.	TOPICS	HOURS	LEARNING OUTCOMES
1 (Oct 14-17)	**Introduction, Delivery of Material etc.**	1	1,2
	Basic Theory of Numbers & Operations 1.1 Basic Theory of Numbers 1.2 Arithmetic Operations and Fundamental Laws 1.2.1 The Four Basic Operations **Online Quiz 1** 1.2.2 Fundamental Laws of Operations 1.3 Directed Numbers and their Operations 1.4 Sequence of Arithmetic Operations **Online Quiz 2** **Worksheet 1-Moodle Online Test 1**	3	
	Set Theory 2.1 Sets, Types of Sets, Subsets, Venn Diagrams **Online Quiz 3**	1	
2 (Oct 21-24)	2.2 Union and intersection of Sets **Online Quiz 4** 2.3 Application of Set Theory Worksheet 2-Moodle Online Test 2	2	2, 3 and 8
	Basic Arithmetic 3.1 Factors and Multiples of a Number 3.1.1 Highest Common Factor (HCF) **Online Quiz 5** 3.1.2 Lowest Common Multiple (LCM) 3.1.3 Application of HCF & LCM **Online Quiz 6** 3.2 Reducing fractions to Simplest form 3.3 Addition and Subtraction of Fractions **Online Quiz 7**	3	

WEEK No.	TOPICS	HOURS	LEARNING OUTCOMES
3 (Oct 27-31)	3.4 Multiplication and Division of Fractions **Online Quiz 8** 3.5 Estimation/Rounding off 3.6 Scientific Notation **Online Quiz 9 & Worksheet 3- Moodle Online Test 3**	2	3, 6 and 8
	Basic algebra (Part-1) 4.1 Power Number Algebra and Laws of Indices **Online Quiz 10** 4.2 Algebra- Use of Symbols & Substitution 4.3. Addition and Subtraction of Polynomials 4.4 Multiplication of Polynomials **Online Quiz 11** **Worksheet 4 – Moodle Online Test 4**	3	
	Revision for CA1		
	Continuous Assessment 1 **(Topics: Units 1, 2 and 3)**		1,2,3,6,8
4 (Nov 3 - 7)	**Techniques of Factorization and Rational Expressions** 5.1 Factorisation of Polynomials **Online Quiz 12** 5.2 Simplification of Rational Expressions 5.3 Multiplication and Division of Rational Expressions **Online Quiz 13** 5.4 Addition and Subtraction of Rational Fractions 5.5 Rationalising denominators of Irrational Expressions **Online Quiz 14** **Worksheet 5 – Moodle Online Test 5**	4	3, 4, 6, 8
	Units of Measurements, Percentages and Ratios 6.1 and 6.1.1 Units of Measurements and Conversions **Online Quiz 15**	1	
5 (Nov 10-14)	6.1.2 Inter-system conversions 6.1.3 Measuring Temperature **Online Quiz 16** 6.2 Percentages **Online Quiz 17** 6.3 Ratio and Proportion 6.3.1 Ratio and 6.3.2 Rate **Online Quiz 18** 6.3.3 and 6.3.4 Direct Proportion and Inverse Proportion 6.4 Map Scales (**Online Quiz 19**) **Worksheet 6-Moodle Online Test 6**	4	3,5,8
	Linear Equations, Inequalities and their Applications 7.1 Solving Linear Equations	1	

WEEK No.	TOPICS	HOURS	LEARNING OUTCOMES
6 (Nov 17-21)	7.2 Simultaneous Linear Equations 7.2.1 Solution by Elimination Method **Online Quiz 20** 7.2.2 Solution by Substitution Method 7.3 Linear inequalities 7.3.1 Methods of describing inequalities **(Online Quiz 21)** 7.3.2 Solving linear inequalities **(Online Quiz 22)** **Worksheet 7 – Moodle Online Test 7**	**3**	**5, 9**
	Modelling Simple Real Life Problems 8.1 Word problems on linear equations 8.2 Word problems on linear inequalities **Online Quiz 23** **Worksheet 8 – Moodle Online Test 8**	**2**	
7 (Nov 24-28)	**Quadratic Equations and Formulas** 9.1 Solving Quadratic equations 9.1.1 and 9.1.2 Solutions by Factorisation and Formula **Online Quiz 24** 9.2 Formation of Quadratic Equations 9.3 Equations involving radicals **(Online Quiz 25)** 9.4 Formula Transposition/subject change in Formula **Online Quiz 26** **Worksheet 9-Moodle Online Test 9**	**3**	**7**
	Revision of Units 4 to 8		
	Continuous Assessment 2 **(Topics: Units 4, 5, 6, 7 and 8)**	**2**	**3,4,5,6,8,9**
8 (Dec 1-5)	**Angles and their measure** 10.1 Types of Angles 10.2 Conversion from radians to degrees and vice-versa **Online Quiz 27** 10.3 Length of an Arc and area of a Sector **Online Quiz 28** **Worksheet 10 – Moodle Online Test 10**	**3**	**10,11,12,13**
	Trigonometry 11.1.1 Definition and Properties of a Right Triangle. 11.1.2 The Pythagoras Theorem **Online Quiz 29**	**2**	

WEEK No.	TOPICS	HOURS	LEARNING OUTCOMES
9 (Dec 8-12)	11.2 Trigonometric ratios in a Right Triangle 11.2.1 The Six Trigonometric Ratios **Online Quiz 30** 11.2.2 Fundamental Trigonometric Identities 11.2.3 Applications of Trigonometric Identities **Online Quiz 31** 11.3 Solutions of Right Triangles and Applications 11.4 Angles of Elevation and Depression **Online Quiz 32** **Worksheet 11- Moodle Online Test 11**	5	11,12,13
10 (Dec 15-19)	**Plane Coordinate Geometry** 12.1 The Rectangular Coordinate System 12.2 Distance between two points 12.3 Gradient or Slope of a line **Online Quiz 33** 12.4 Equation of a straight line 12.5 Drawing graph of the straight line function based on its equation 12.6 Parallel and perpendicular lines **Online Quiz 34** **Worksheet 12 – Moodle Online Test 12**	5	14
11 (Dec 22-26)	**The Circle & Symmetry** 13.1 Centre and radius of the circle, tangent lines **Online Quiz 35** 13.2 Symmetry of Graphs **Worksheet 13 -Online Test Online Test 13** **Revision of Units 9 – 13**	5	14,15
12 (Dec 29 – Jan 2)	**Final Exam (Topics: Units 9, 10, 11, 12, 13)**		
	TOTAL NUMBER OF TEACHING HOURS	55	

Indicative Reading		
Title/Edition/Author	**Publisher**	**ISBN**
College Algebra with Trigonometry, (9th Edition 2010), Raymond A. Barnett, Michael Ziegler and Karl Byleen, David Sobecki,	McGraw Hill	9780077350109
Basic Engineering Mathematics, (8th Edition 2021), Bird J.	Routledge	9780367643676
Engineering Mathematics, (8th Edition 2020), Stroud K.A and Booth D.J,	Red Globe Press	9781352010275

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Contents

(Unit 9) Quadratic Equations and formulas

9.1 Solving Quadratic Equations

Any expression of the form $ax^2 + bx + c$ where $a \neq 0$ is called a quadratic expression.

Examples:

- $x^2 + 2x + 1$
- $2x^2 + 4$
- $x^2 - 9$
- $3x^2 + 15$

The moment we put an equal sign " = " , in front of a quadratic expression it becomes a **quadratic equation** $ax^2 + bx + c = 0$

Examples of quadratic equations are:

- $2x^2 + 3x - 2 = 0$
- $3x^2 - 5x = 0$
- $3x^2 = 27$
- $x^2 - 5x + 6=0$

There are many ways of solving quadratic equations. Among these we have

- Solution by factorisation
- Solution by quadratic formula

We shall focus on solution by **factorisation** and solution by **formula**.

9.1.1 Solutions by factorization

There are three (3) steps in using this method:

i) Write the quadratic equation in the standard form $ax^2 + bx + c = 0$.

ii) Factorise the quadratic expression on the left side of the equation.

iii) Solve by equating each of the factors to zero (0).

Example 1: Solve: $3x^2 - 5x = 0$

Solution:

$x(3x - 5) = 0$ (factorising the left side)

For two numbers to give a product of zero (0) one of them must be equal to zero,

Hence $\boldsymbol{x = 0}$ or $3x - 5 = 0$

$3x = 5 \Rightarrow \boldsymbol{x = \frac{5}{3}}$

Example 2: Solve: $3x^2 = 27$

Solution: Dividing by 3, we get $x^2 = 9$

$x^2 - 9 = 0$ (taking 9 to LHS)

$(x - 3)(x + 3) = 0$ (factorising)

$x - 3 = 0$ or $x + 3 = 0$

$\therefore$ $\boldsymbol{x = 3}$ **or** $\boldsymbol{x = -3}$

Example 3: Solve: $x^2 - 5x + 6 = 0$

Solution: Any method can be used to factorise the expression on the left hand side of the " = " sign.

Thus factorising gives $(x - 2)(x - 3) = 0$

Hence $x - 2 = 0$ or $x - 3 = 0$

$\boldsymbol{x = 2}$ or $\boldsymbol{x = 3}$

Example 4: Solve $2x^2 + 3x - 2 = 0$

Solution:

$2x^2 + 3x - 2 = 0 \Rightarrow 2x^2 + 4x - x - 2 = 0$

$(2x^2 + 4x) - (x + 2) = 0$

$2x(x + 2) - 1(x + 2) = 0$

$(2x - 1)(x + 2) = 0$

$2x - 1 = 0$ or $x + 2 = 0$

$2x = 1$ or $x = -2$

$\therefore$ $\boldsymbol{x = \frac{1}{2}}$ or $\boldsymbol{x = -2}$

Note: Trial and Error method can also be used to factorise any quadratic expressions given on the left hand side of a quadratic equation.

It is not always possible to solve quadratic equations by factorisation and it can also take a lot of time to complete the square. The quadratic formula is an easier way to solve such type of equations.

Link: Solution by factorisation

9.1.2 Solutions by formula

Solution of $ax^2 + bx + c = 0$ is given by

$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$, where a, b and c are constants, with $a \neq 0$.

Example 1: Solve $x^2 - 5x - 3 = 0$

Solution: Comparing with $ax^2 + bx + c = 0$, we have $a = 1, b = -5$ and $c = -3$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(-3)}}{2(1)}$$

$$x = \frac{5 \pm \sqrt{25 + 12}}{2}$$

$$x = \frac{5 \pm \sqrt{37}}{2}$$

$$x = \frac{5 \pm 6.08276}{2}$$

$$x = \frac{5 + 6.08276}{2} = 5.54138$$

or

$$x = \frac{5 - 6.08276}{2} = -0.54138$$

$\boldsymbol{x = 5.54}$ ***or*** $\boldsymbol{x = -0.54}$ (to 2 decimal places)

Example 2: Solve $x^2 - 3x - 2 = 0$

Solution: Comparing with $ax^2 + bx + c = 0$, we have $a = 1, b = -3$, and $c = -2$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4 \times 1 \times -2}}{2 \times 1}$$

$$x = \frac{3 \pm \sqrt{9 + 8}}{2} = \frac{3 \pm \sqrt{17}}{2}$$

$\boldsymbol{x = 3.56}$ **or** $\boldsymbol{-0.56}$ (to 2 decimal places)

Link: Solution by formula

Class Activity 9.1

Solve the following equations:

(1) Using factorisation method

i) $x^2 + 8x = -15$

ii) $x^2 - 4 = 0$

iii) $3x^2 - 8 = 10x$

2. Using quadratic formula and leaving your answers correct to 2 decimal places, solve

i) $x^2 - 4x + 2 = 0$

ii) $5x^2 + 3x - 3 = 0$

9.2 Formation of Quadratic Equations

Sometimes we may be given the **solutions** or **roots** of a quadratic equation and then be required to find the equation where those roots belong.

To form a quadratic equation, let $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ be the two roots which implies $x = \alpha$ or $x = \beta$, then we write the equation as:

$(x-\alpha)(x-\beta) = 0$. If we expand the left hand side, we get: $x^2 - \alpha x - \beta x + \alpha\beta = 0$. Combining similar terms, the final equation is:

$\boldsymbol{x^2 - (\alpha+\beta)x + \alpha\beta = 0}$. ……(1)

$$x^2 - (\text{sum of the roots})x + (\text{product of the roots}) = 0$$

$x^2 - Sx + P = 0$, where $S =$ sum of the roots and $P =$ product of the roots............... (1)

The Formula (1) above is used for the formation of a quadratic equation when its roots are given.

Example 1: Find the quadratic equation whose roots are 5 and -2.

Solution: Here $\alpha = 5$ and $\beta = -2$

$$\alpha + \beta = 3 \text{ and } \alpha\beta = -10$$

Using equation: $x^2 - (\alpha+\beta)x + \alpha\beta = 0$ above the equation with given roots is

$$\boldsymbol{x^2 - 3x - 10 = 0}$$

Example 2: Find the quadratic equation whose roots are 0 and 5.

Solution: Here $\alpha = 5$ and $\beta = 0$

$$\alpha + \beta = 5 \text{ and } \alpha\beta = 0$$

Using Equation: $x^2 - (\alpha+\beta)x + \alpha\beta = 0$

$$\therefore \quad x^2 - 5x = 0$$

Example 3: Find the quadratic equation whose roots are $\frac{1}{4}$ and -4.

Solution: Here $\alpha = \frac{1}{4}$ and $\beta = -4$

$\alpha + \beta = \frac{1}{4} - 4 = -\frac{15}{4}$ and $\alpha\beta = -1$

Using Equation: $x^2 - (\alpha+\beta)x + \alpha\beta = 0$

$$x^2 - \left(-\frac{15}{4}\right)x - 1 = 0$$

$$x^2 + \frac{15}{4}x - 1 = 0$$

$$4x^2 + 15x - 4 = 0$$

Class Activity 9.2

1. If the sum of the roots is 8, and product of the roots is 15, find the quadratic equation.
2. Find the quadratic equation whose roots are $x = 2$ and $x = 3$.
3. Find the quadratic equation whose roots are $x = -2$ and $x = -3$.
4. Find the quadratic equation whose roots are $x = 0$ and $x = -4$.

9.3 Equations involving radicals

A radical equation is an equation in which the variable is inside a radical symbol (in the radicand).

Examples:

- $\sqrt{x} - 4 = 0$
- $\sqrt{(x-1)} - 2 = 3$

Note: $x + \sqrt{5} = 10$ is not a radical equation.

Example 1: Solve: $\sqrt{(x-1)} = 3$

Solution:

$$\sqrt{(x-1)} = 3$$

Squaring both sides gives

$$x - 1 = 9$$
$$x = 10$$

Example 2: Solve: $\sqrt{(3x+1)} + 3 = 7$

Solution: Isolating the radical term:

$$\sqrt{(3x+1)} = 7 - 3$$
$$\sqrt{(3x+1)} = 4$$

Squaring both sides gives

$$3x + 1 = 16$$
$$3x = 15$$
$$x = 5$$

Example 3:
Solve: $\sqrt{(5x+1)} = \sqrt{(6x-2)}$

Solution: Squaring both sides

$$5x + 1 = 6x - 2$$
$$5x - 6x = -2 - 1$$
$$-x = -3$$
$$x = 3$$

Class Activity 9.3

(1) Solve: $\sqrt{x} = 2$

(2) Solve: $\frac{1}{\sqrt{x}} = \frac{1}{\sqrt{5}}$

(3) $3\sqrt{(x-2)} = 2\sqrt{(x+8)}$

(4) Solve: $\sqrt{(9-x)} + 3 = 5$

(5) Solve: $\sqrt{x} = 2\sqrt{x-1}$

9.4 Formula transposition/subject change in formula

A mathematical ***equation*** is a statement indicating that two quantities are equal. The left hand side and right hand side of the equation are connected by the equal sign " $=$ ". The left hand side is called the subject of the equation.

An example of an ***equation*** is the linear equation which is in the form: $ax + b = 0$ where a and b are constants and a is not zero.

A ***formula*** is a special type of equation that shows the relationship between different **variables**.

For example: $P = 2L + 2W$ which is the formula for finding the *perimeter* of a rectangle.

A **variable** is a symbol like the L and the W that stands for a number we don't know yet.

The term **transposition** means changing the position of a variable or any term from one side of the equation to another.

To manipulate any equation, the following properties of equality are used:

(i) Addition Property of Equality (APE)
If $x + a = b$, then we can add (-a) to both sides of the equation to isolate x. Thus,
$x + a + (-a) = b + (-a)$
$x = b - a$

Note: When $+a$ is transposed to the right side, it becomes $-a$.

(ii)Multiplication Property of Equality (MPE)
If $ax = b$ then we can multiply both sides by $\frac{1}{a}$ to get the value of x. Thus,
$\frac{ax}{a} = \frac{b}{a}$ then $x = \frac{b}{a}$.

(iii) Symmetric Property of Equality
If $a = x$, then $x = a$.

Example 1: Given the formula, $I = \frac{PRT}{100}$, make P the subject of the formula.

Solution: $I = \frac{PRT}{100}$

Multiplying the whole equation by 100 to remove fractions, we get:

$$100\, I = PRT$$

Dividing both sides by RT, then

$$P = \frac{100\, I}{RT}$$

Example 2: Solve for x in the equation:

$$y = \frac{a+x}{5-bx}$$

Solution: Multiplying both side with $(5 - bx)$, we have: $y(5 - bx) = (a + x)$

$$5y - bxy = a + x$$

Bring the terms with the variable x to one side

$$5y - a = x + bxy$$

Factorizing x, we get: $5y - a = x(1 + by)$

Dividing both sides by the expression $(1 + by)$, we get $\frac{5y-a}{1+by} = x$

Hence $x = \frac{5y-a}{1+by}$ makes x the subject.

Example 3: Make g, the subject of the formula: $T = 2\pi\sqrt{\frac{l}{g}}$

Solution: We begin by squaring both sides of the equation in order to remove the square root.

$$T^2 = (2\pi)^2 \frac{l}{g}$$

To remove the fraction we multiply both sides by g,

$$T^2 g = (2\pi)^2 l$$

Dividing both sides by T^2 gives $g = \frac{(2\pi)^2 l}{T^2}$

Class Activity 9.4

Work on each equation or formula to make the variable indicated the subject of the formula.

1) $3k = kx + 5$ make "x" the subject.

Solution:

2) $y = \frac{7}{4+x}$, make "x" the subject.

Solution:

3) $K = \frac{3n+2}{n+3}$, make "n" the subject.

Solution:

4) $E = \frac{mv^2}{2}$, make "m" the subject.

Solution

5) $E = \frac{m^2v}{2g}$, make "g" the subject.

Solution:

6) $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$, make "$u$" the subject.

Solution:

Worksheet-9

1. What is the value of x in the equation $\frac{x+3}{x-3} - \frac{4x^2}{x^2-9} = \frac{x-4}{x+3}$?

a). $x = 0 \; or \; x = 3$

b). $x = \pm 3$

c). $x = \frac{1}{4}$

2. What is the value of ***k*** so that the quadratic equation $x^2 + kx + 25 = 0$ will have two identical roots?

a). $k = \pm 10$

b). $k = 5$

c). $k = -5$

3. The area of a rectangle is 85 sq.cm and its length is 2 cm more than 3 times its width. What is the width of the rectangle?

a). $W = 5 \; cm$

b). $W = 12 \; cm$

c). $W = 17 \; cm$

4. Two roots of a quadratic equation have a sum of 8 and a difference of 4. What is the equation?

a). $x^2 - 4x + 8 = 0$

b). $x^2 - 8x + 12 = 0$

c). $x^2 - 4x + 6 = 0$

5. What is the quadratic equation that has an irrational root which is $2 + \sqrt{3}$?

a). $x^2 - 4x + 8 = 0$

b). $x^2 - 4x - 5 = 0$

c). $x^2 - 4x + 1 = 0$

6. What is the value of ***x*** that satisfies the equation $\sqrt{4x-2} = \sqrt{2x}$?

a). $x = 2$

b). $x = -1$

c). $x = 1$

7. If $16 = r^2 + rh$, which formula is correct to solve for r in terms of h?

a). $r = \frac{16}{r + h}$

b). $r = \frac{-h \pm \sqrt{h^2+64}}{2}$

c). $r = \sqrt{16 - rh}$

8. From $x = \frac{y+2}{y-2}$, make y as the subject. Which equation is correct to solve for y?

a). $y = \frac{2+2x}{x-1}$

b). $y = \frac{2x-2}{x+1}$

c). $y = xy + 2x - 2$

9. The formula for degrees Celsius is given by either $C = \frac{5}{9}\ (F - 32)$ or $C = K - 273.15$. If K is solved in terms of F, which of the following is correct?

a). $K = \frac{5}{9}F + 305.15$

b). $K = \frac{5}{9}F + 255.37$

c). $K = \frac{9}{5}(F - 32) - 273.15$

10. **Solve by factorising**

i) $x^2 = 9$

ii) $4x^2 - 12x = 0$

iii) $x^2 + 5x - 6 = 0$

iv) $x^2 - 9x + 20 = 0$

v) $3x^2 - 7x - 6 = 0$

11. **Solve by quadratic formula leaving answers correct to 2 decimal places**:

$2x^2 + 5x - 4 = 0$

12. Find the quadratic equation whose roots are $\boldsymbol{x} = -\frac{\mathbf{1}}{\mathbf{2}}$ and $x = \frac{1}{2}$

13. Find the quadratic equation whose roots are $x = -4$ twice.

14. Solve: $\sqrt{(y+3)} - 1 = 7$

15. Solve: $\frac{\sqrt{x}}{2} = \sqrt{\frac{(3x-14)}{5}}$

16. Make t the subject of the formula:

$v = u + \text{at}$

17. Make a the subject of the formula:

$s = ut + \frac{1}{2}at^2$

18. Make y the subject of the formula:

$\frac{y}{y+x} + 5 = x$

19. Make x the subject of the formula:

$y(2x+1) = x+1$

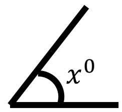
20. Make r the subject of the formula:

$P = \frac{P_0}{1-r^2}$

(Unit-10) Angles and Their Measure

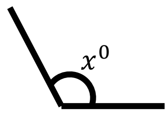
10.1 Types of Angles

1. Acute Angle



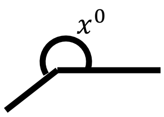
$x^0 < 90^0$

2. Obtuse Angle



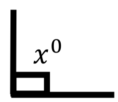
$90^0 < x^0 < 180^0$

3. Reflex Angle



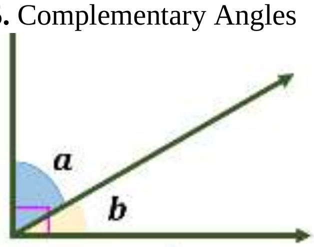
$180^0 < x^0 < 360^0$

4. Right Angle



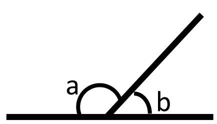
$x^0 = 90^0$

5. Complementary Angles



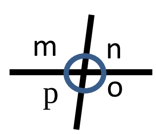
$m\angle a + m\angle b = 90°$

6. Supplementary Angles



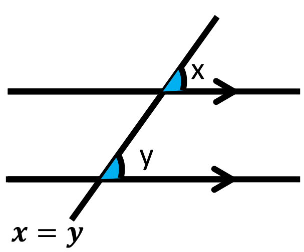
$a + b = 180^0$

7. Angle at a Point



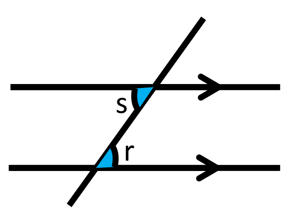
$m + n + o + p = 360^0$

8. Corresponding Angles



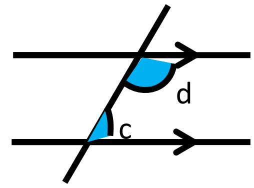
$x = y$

9. Alternate Angles



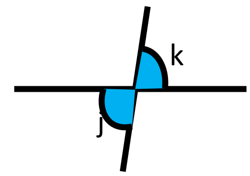
$r = s$

10. Allied Angles



$c + d = 180^0$

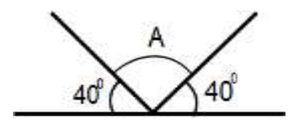
11. Vertically Opposite Angles



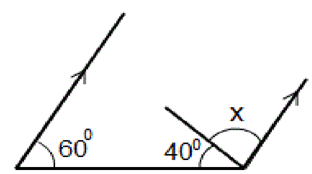
$j = k$

Class Activity 10.1

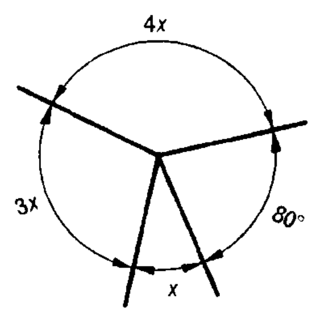
1. **Find A**



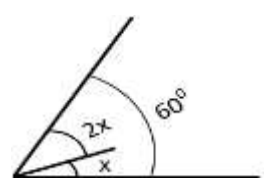
2. **Find x.**



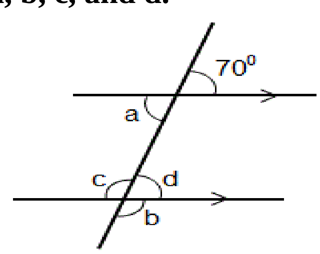
3. **Find x.**



4. **Find x.**

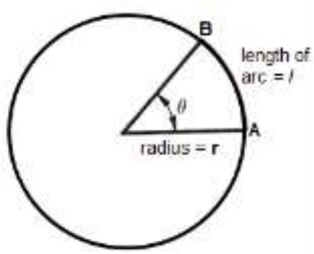


5. **Find a, b, c, and d.**



10.2 Conversions : Degree ↔Radian

Radian: A radian is defined as the angle subtended at the centre of a circle by an arc equal in length (l) to the radius of the circle (r) as shown in following figure.



Angle in radians$(\theta) = \frac{\text{Length of Arc}(l)}{\text{Radius of Circle}(r)}$

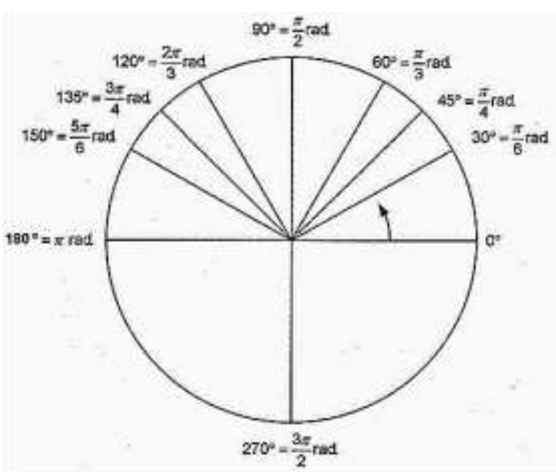
Note:

(1) $\boldsymbol{\pi\, rad = 180^0}$

(2) $\boldsymbol{1\ degree = \frac{\pi}{180}\ radians}$

(3) $\boldsymbol{1\ radian = \frac{180}{\pi}\ degrees}$

Comparison of degree and radian measure



Example 1: Find the angle in radians subtended by an arc 12.9 cm long whose radius is 4.6 cm.

Solution:

Angle in radians$(\theta) = \frac{\text{Length of Arc}(l)}{\text{Radius of Circle}(r)}$

$\theta = \frac{12.9}{4.6} = 2.804$ radians (up to 3 decimals)

Example 2: Express an angle of 104° in radians.

Solution:

$\theta = \frac{\pi\times\theta^\circ}{180} = \frac{\pi(104^\circ)}{180} = 1.815$ radians

Example 3: Express the angle of $\frac{\pi}{4}$ in degrees.

Solution: $\boldsymbol{\theta} = \frac{\mathbf{180^\circ}}{\boldsymbol{\pi}} \times \frac{\pi}{4} = 45^\circ$

Class Activity 10.2

1) Find the radian measure of a central angle θ opposite an arc of length, $\boldsymbol{l} = 15\ m$, in a circle of radius $r = 3m$.

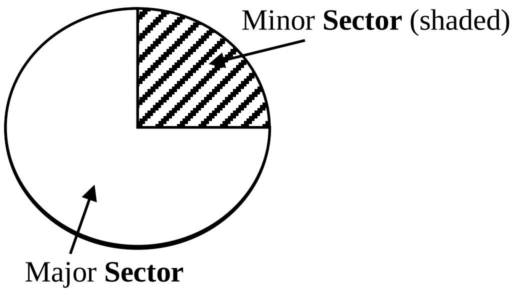
2) The angle $\frac{3\pi}{4}$ in degrees is …
(a) 60°
(b)120°
(c) 135°

3) The angle 120° in radians is …
(a) $\frac{\pi}{3}$
(b) $\frac{2\pi}{3}$
(c) $\frac{3\pi}{4}$

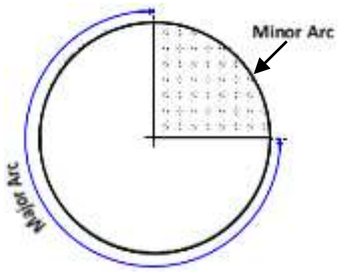
4) If one complete rotation corresponds to 360° then the degree measure of $\frac{7}{6}$ rotations is …
(a) 180°
(b) 210°
(c) 420°

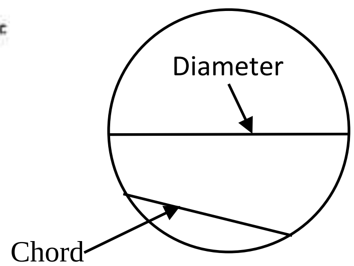
10.3 Length of an Arc and Area of a Sector

Important parts of a circle



a) **Major Sector** is the bigger unshaded portion and **minor sector** is the smaller shaded portion in the diagram above.



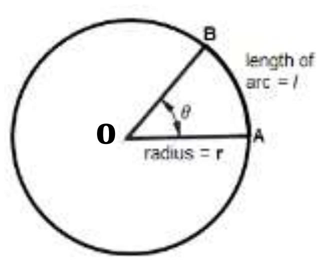


b) The **major arc** is the longer part of the circumference, while the **minor arc** is the shorter part.

c) **Diameter** is the longest chord dividing the circle into two equal parts. However a chord which does not pass through the centre is not a diameter.

10.3.1 Length of an Arc

From the previous definition of radian we had



$$\text{Angle in radians}(\theta) = \frac{\text{Length of Arc}(l)}{\text{Radius of Circle}(r)}$$

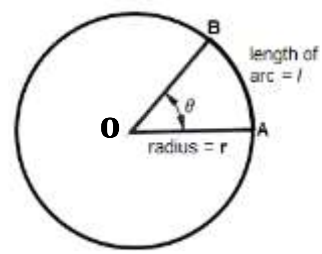
When we make length of an arc(l) the subject, we have: $\boldsymbol{l = r\theta}$, where θ is in radians.

Example 1: Find the length of an arc of a circle of radius 5.5 cm when the angle subtended at the centre is 1.20 radians.

Solution: From the equation $l = r\theta$,
Arc length, $l = (5.5)(1.20) = \mathbf{6.60\ cm}$

Example 2: Determine the diameter of a circle if an arc of length 4.75 cm subtends an angle of 0.91 radians.
Solution: Since arc length, $l = r\theta$,
making r the subject we have:
$r = \frac{l}{\theta} = \frac{4.75}{0.91} = 5.22\ cm$
Diameter $= 2 \times$ radius
$= 2 \times 5.22 = \mathbf{10.44\ cm}$

10.3.2 Area of a Sector



From the diagram above consider the minor sector AOB, with angle θ as indicated.
To get the area of the sector we have

a) Area of sector, $\boldsymbol{A} = \frac{\boldsymbol{\theta}}{\mathbf{360}}(\boldsymbol{\pi r^2})$, when θ is in degrees.

b) When θ is in radians then,

$$\boldsymbol{A} = \frac{\theta}{2\pi}(\pi r^2) = \frac{\mathbf{1}}{\mathbf{2}}\boldsymbol{r^2\theta}.$$

Link: [Arc length & area of a circle]

Example 1: Determine the area of the minor sector formed when an angle of 1.71 radians is subtended at the center of a circle with diameter 42 cm.

Solution: Area of a sector, $A = \frac{1}{2}\boldsymbol{r^2\theta}$,
$\boldsymbol{radius, r = half of}$ **diameter** $= \mathbf{21\ cm}$
$\therefore A = \frac{1}{2}(21)^2(1.71) = \mathbf{377.055\ cm^2}$

Class Activity 10.3

1. State whether each statement below is **True** or **False.**
   a) All chords are diameters (True/False)
   b) All diameters are chords(True/False)
   c) An arc is a part of the circumference
   (True/False)

2. Zaid is driving his car, enters a roundabout and maintains a fixed distance of 10 meters from the centre. When he has covered an arc length of 50 meters, how much more radians he needs to turn through to complete 1 rotation?
   a) $2\pi \ radians$
   b) $1.28 \ radians$
   c) $5 \ radians$

3. Calculate:

i) The length of an arc of a circle of radius 8.32 cm, when the angle subtended at the centre is 2.14 radians.

ii) The area of the minor sector formed.

4. If the angle subtended at the centre of a circle of diameter 82 mm is 1.46 rad, find the length of the major arc.

Worksheet 10

For questions 1 to 8 circle the correct answers.

1. Angles ***A*** and ***B*** are complementary. Angle ***B*** is $15°$ more than twice the measure of angle ***A***. The measure of angle ***B*** is …
   a) $65°$
   b) $75°$
   c) $55°$

2. The angle $\frac{3\pi}{2} - \frac{2\pi}{3} + \frac{3\pi}{4}$ in degrees is …
   a) $180°$
   b) $270°$
   c) $285°$

3. The angle $\frac{2\pi}{3}$ in degrees is …
   (a) 60^0
   (b) 120^0
   (c) 135^0

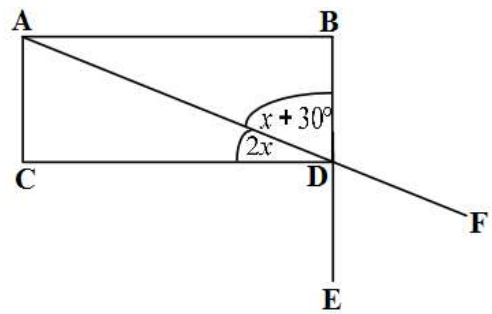
4. The angle $300°$ in radians is …
   (a) $\frac{5\pi}{3}$
   (b) $\frac{2\pi}{3}$
   (c) $\frac{3\pi}{4}$

5. The angle in radians subtended by an arc 15cm long whose radius is 30mm is …
   (a) 5
   (b) 0.5
   (c) 1/5

6. Two radii of a circle form a central angle of $210°$ when they intercept an arc with length $8cm$. The radius of the circle is …
   (a) 2.18 cm
   (b) 0.04 cm
   (c) 1680 cm

7. Two angles are supplementary. When the smaller angle is increased by $20°$, it becomes one-third the measure of the larger angle. The smaller angle is …
   a) $50°$
   b) $60°$
   c) $30°$

8. What is the measure of angle EDF in the figure below if ABCD is a rectangle?



   a) $20°$
   b) $50°$
   c) $40°$

Show all working steps for the following questions:

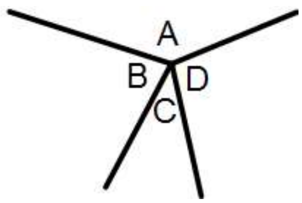
9. Convert the following angles from degrees to radians in terms of π.
   (i) $315°$
   (ii) $210°$
   (iii) $225°$

10. Convert the following angles from radians to degrees.
   (i) $\frac{-\pi}{3}$
   (ii) $\frac{3\pi}{2}$
   (iii) $\frac{2\pi}{5}$

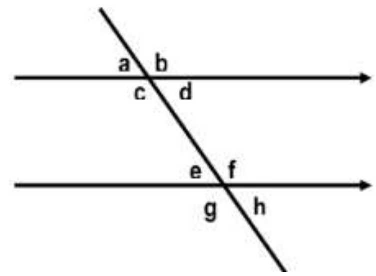
11. Determine the length of the radius and circumference of a circle if an arc length of 32.6 cm subtends an angle of 3.76 radians.

12. A football stadium flood light can spread its illumination over an angle of 45° to a distance of 55 m. Determine the maximum area that is floodlit.

13. In the following figure, if $\angle B = 80^0$, $\angle C = 40^0$ and $\angle D = 100^0$ then $\angle A$ is _____



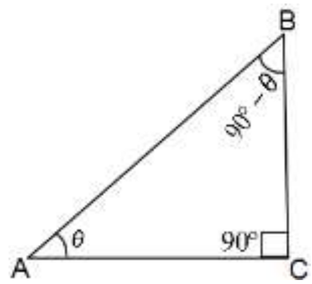
14. In the following figure, if angle $d = 60°$ find the angles marked $a, c, e, and\ g$.



15. A minor arc of a circle subtends an angle of 0.785 radians. If the radius of the circle is 15cm, what is the length of the major arc?

(Unit-11) Trigonometry

11.1 Definition and Properties of a Right Triangle.



The triangle above is called a right triangle.
A **right triangle** (American English) or **right – angled triangle** (British English) is a triangle in which one angle is a right angle (that is, a 90°-degree angle). The relation between the sides and angles of a right triangle is the basis for trigonometry.

In the given right triangle, the following statements are true:

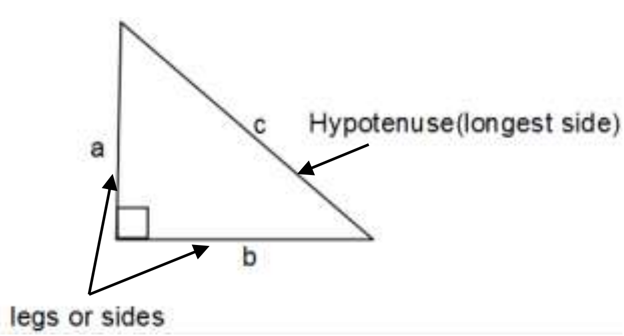
i) Angles A and B are both acute angles, each of them measures less than 90 degrees.

ii) Angles A and B are complementary angles such that $\mathbf{A + B = 90°}$.

iii) The sum of the angles A, B and C is $\mathbf{180°}$, so $\mathbf{A + B + C = 180°}$

11.1.2. The Pythagoras Theorem

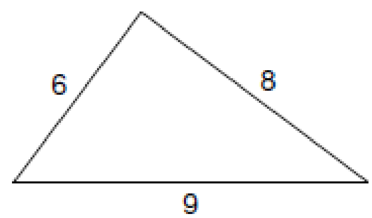
In mathematics, **the Pythagorean Theorem**, also known as **Pythagoras' theorem**, is a fundamental relation in Euclidean geometry among the three sides of a right triangle. It states that ***the square of the hypotenuse (the side opposite the right angle) is equal to the sum of the squares of the other two sides.***



A **Pythagorean triple** consists of three positive integers a, b, and c, such that $a^2 + b^2 = c^2$. Hence a right triangle exists with two legs (perpendicular sides) and a hypotenuse.

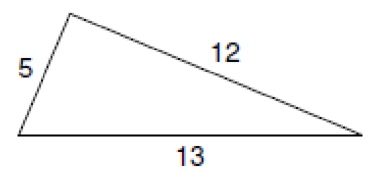
Pythagorean triples describe the three sides of a right triangle.

Example 1: Is the given triangle a right angled triangle?



Solution:
Since $9^2 \neq 8^2 + 6^2$, so it is not a right angled triangle.

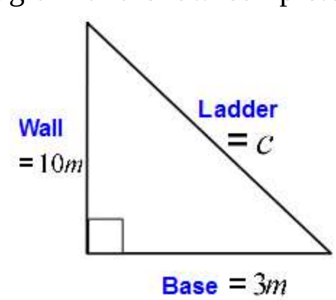
Example 2: Does the following triangle form a right angled triangle?



Solution: Since $13^2 = 12^2 + 5^2$, so it is a right angled triangle.

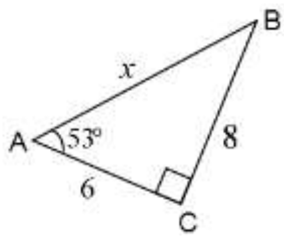
Example 3: A firefighter places the ladder against the side of a **10*m*** house. If the base of the ladder is 3*m* away from the house, how long is the ladder? Draw a diagram and show complete solution.

Solution:
$c^2 = 10^2 + 3^2$
$c = \sqrt{109} = 10.44$
∴Length of ladder is 10.44*m*



Class Activity 11.1

1) Given the right triangle below:

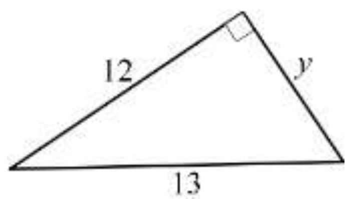


Find the following:

a) value of angle B

b) value of side *x*.

2) Find side *y* in the following triangle:



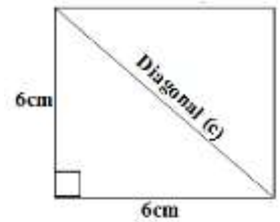
3) Determine whether the following integers form a **Pythagorean triple** or not.

i) (15, 8, 17)

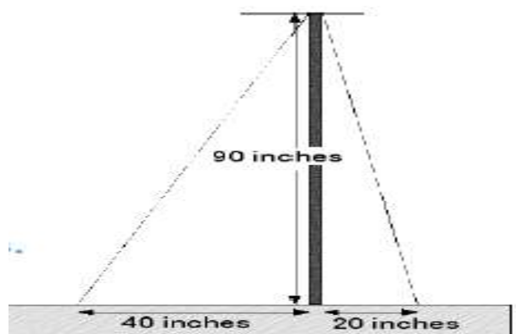
ii) (12, 6, 8)

4) Calculate the length of the diagonal of a square with sides of length 6*cm*.

Solution:

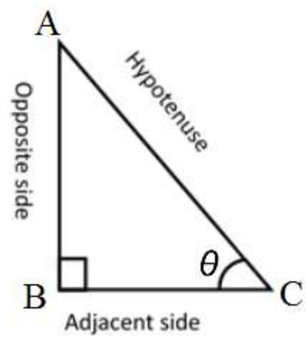


5) What length should the wires be to hold the pole of 90 inches at a right angle to the roof if two wires are fixed 40 inches and 20 inches away from the foot of the pole?



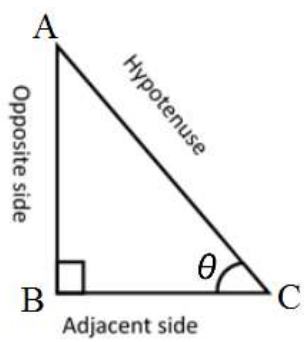
11.2 Trigonometric Ratios in a Right Triangle

In defining the six trigonometric functions of a particular acute angle, let us consider the figure below in which θ degrees is the measure of the given angle. The **opposite side** and the **adjacent side** are always relative to the position of the angle θ.



In any right-angled triangle, we can establish the relationships between the sides and angles of the triangle. Some of these relations are defined by means of trigonometric ratios: Sine, Cosine and Tangent.

11.2.1 Sine of an angle

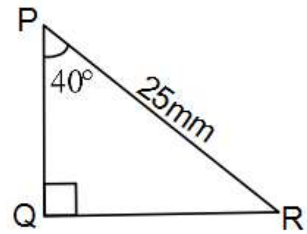


The Sine of an angle = $\dfrac{\textbf{Opposite side}}{\textbf{Hypotenuse}}$

$\sin\theta = \sin C = \frac{AB}{AC}$ and $\sin A = \frac{BC}{AC}$

Example 1: Given right triangle PQR in which angle P = 40^0, side PR = 25*mm*, find the length of side QR. (*Round off your answer to 2 decimals*).

Solution:



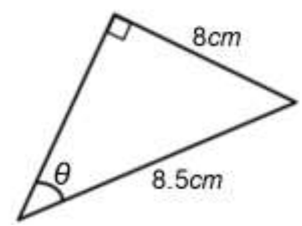
$$\sin\ P = \frac{QR}{PR}$$

$$\sin\ 40° = \frac{QR}{25}$$

$QR = 25\sin 40° = 16.07mm$ (to 2 decimals).

Example 2:
Find the angle marked θ in the right-angled triangle below.
(Round off your answer to 2 significant figures).

Solution:

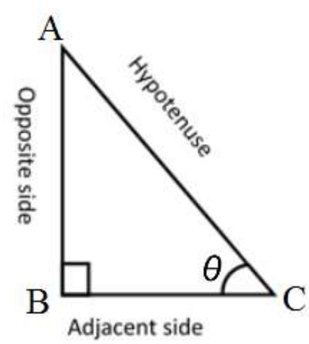


$$\sin\theta = \frac{8}{8.5}$$

$$\Rightarrow \theta = sin^{-1}\left(\frac{8}{8.5}\right)$$

$\therefore \theta = 70°$ (*to 2 significant figures*)

11.2.2 Cosine of an angle

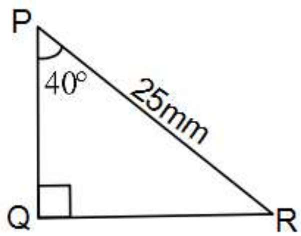


The Cosine of an angle = $\dfrac{\textbf{Adjacent side}}{\textbf{Hypotenuse}}$

$\cos\theta = \cos C = \dfrac{\text{BC}}{\text{AC}}$ and $\cos A = \dfrac{\text{AB}}{\text{AC}}$

Example 1: Given right triangle PQR in which angle P = 40^0, hypotenuse = PR=25*mm*, find the length of side PQ. (Round off your answer to 2 decimals).

Solution:

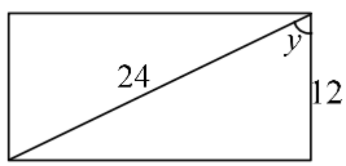


$$\cos\ P = \frac{PQ}{PR}$$

$$\cos\ 40° = \frac{PQ}{25}$$

$PQ\ = 25 \cos 40° \ = 19.15mm$(up to 2 decimals)

Example 2: Find angle *y* in the figure given below.



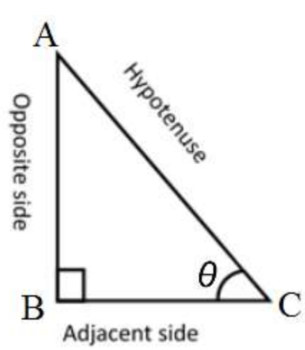
Solution: $cos\ y\ = \frac{12}{24},\quad y = \ cos^{-1}(0.5)$

$$y\ = 60°$$

11.2.3 Tangent of an angle

The tan of an angle = $\frac{\textbf{Opposite side}}{\textbf{Adjacent side}}$

$\tan\theta = \tan C = \frac{\text{AB}}{\text{BC}}$ and $\tan A = \frac{\text{BC}}{\text{AB}}$



Example 1: Find angle C, in right triangle ABC, given that AB = 4*cm* and BC = 5*cm.* (Round off your answer to 1 decimal).

Solution:

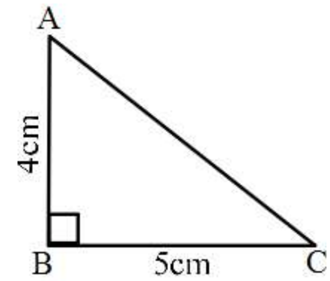
$\tan C = \frac{AB}{AC}$

$\tan C = \frac{4}{5} = 0.8$

$C\ = tan^{-1}(0.8)$

$C\ \ = 38.7°$

(up to 1 decimal place)



Example 2: Given that in a right-angled triangle PQR, angle P = 50^0, QP =7*cm*, find side QR.

Solution:

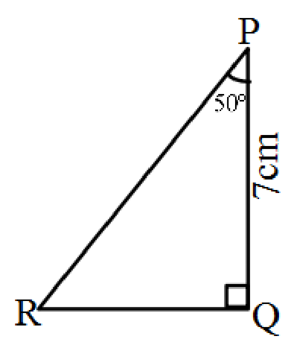
$\tan P = \frac{QR}{QP}$

$\tan 50° = \frac{QR}{7}$

$QR\ = 7 \tan 50°$

$QR\ = 8.34\ cm$

(up to 2 decimals)



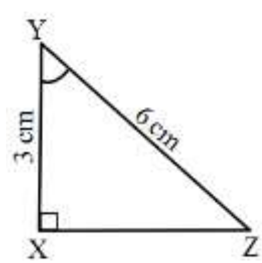
Class Activity 11.2.1

1) Given right-angled triangle PQR, where side PR = hypotenuse = 13*cm* and angle P = 65^0. Find the length of side QR. (Round off your answer to 2 decimals).

2) Given right-angled triangle PQR, where hypotenuse is PR, side PQ = 7*cm*, and angle R=70^0. Find the length of side QR. (Round off your answer to 2 decimal places)

3) In a right-angled triangle ACB, side BC = 5*cm*, and side AC =10*cm*. Find the measure of angle B. (Round off your answer to 1 decimal).

4) In a right triangle YXZ, side XY = 3*cm*, YZ =6*cm*, as shown. Find the angle Y.



11.2.4 Cotangent, Secant and Cosecant of an Angle

The other three (3) ratios are defined as follows:

Cotangent (cot) of an angle = $\frac{\textit{adjacent side}}{\textit{opposite side}}$ $= \frac{1}{\textit{tangent of the angle}}$

Secant (sec) of an angle = $\frac{\textit{hypotenuse}}{\textit{adjacent side}}$ $= \frac{1}{\textit{cosine of the angle}}$

Cosecant (csc) of an angle = $\frac{\textit{hypotenuse}}{\textit{opposite side}}$ $= \frac{1}{\textit{sine of the angle}}$

Link: The six trigonometric ratios

11.2.5 Fundamental Trigonometric Identities

Definition: **Trigonometric identity** is an equation involving trigonometric functions which is true for all permissible values of the angle. We commonly denote the angle with symbol $\boldsymbol{\theta}$ but any letter or symbol can be used for it.

I. Reciprocal Relations

1) $csc\ \theta = \frac{1}{sin\ \theta}$ $\quad sin\ \theta = \frac{1}{csc\ \theta}$

2) $sec\ \theta = \frac{1}{cos\ \theta}$ $\quad cos\ \theta = \frac{1}{sec\ \theta}$

3) $cot\ \theta = \frac{1}{tan\ \theta}$ $\quad tan\theta = \frac{1}{cot\ \theta}$

II. Pythagorean Identities

4) $sin^2\theta + cos^2\theta = 1$

5) $1 + tan^2\theta = sec^2\theta$

6) $1 + cot^2\theta = csc^2\theta$

III. Quotients of Sine and Cosine

7) $\frac{sin\ \theta}{cos\ \theta} = tan\ \theta$

8) $\frac{cos\ \theta}{sin\ \theta} = cot\ \theta$

11.2.6 Applications of Trigonometric Identities

Example 1. If θ is an acute angle in a right triangle and $\sin\theta = \frac{5}{13}$, find the values of the following:

a) $\cos\theta$

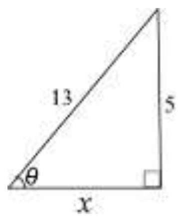
b) $\tan\theta$

c) $\cot\theta$

d) $\sec\theta$

Solution:

a) $\cos\theta$

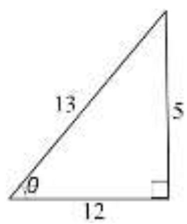


If the data is shown in the triangle above, we use **Pythagoras theorem** to get the value of the unknown side labelled x, in the triangle above.

$x^2 + 5^2 = 13^2$

$x = \sqrt{13^2 - 5^2}$

$x = 12$



The value of x can be can now be substituted and we continue.

Since $\cos\theta = \frac{adjacent\ side}{hypotenuse}$

$\therefore \cos\theta = \frac{12}{13}$

b) $\tan\theta$ [Use: $\tan\theta = \frac{\sin\theta}{\cos\theta}$]

$\boldsymbol{\tan\theta} = \frac{5}{13} \Big/ \frac{12}{13} = \frac{5}{12}$

c) $\cot\theta$ [Use: $\cot\theta = \frac{1}{\tan\theta}$]

$\boldsymbol{\cot\theta} = \frac{12}{5}$

d) $\sec\theta$ [Use: $\sec\theta = \frac{1}{\cos\theta}$]

$\boldsymbol{\sec\theta} = \frac{13}{12}$

Class Activity 11.2.2

1) If θ is an acute angle in a right triangle in which $\tan\theta = \frac{4}{3}$ and $\cos\theta = \frac{3}{5}$, find the values of the following:

a) $\sin\theta$

b) $\cot\theta$

2) Simplify the following expressions:

a) $\tan\theta\cos\theta$

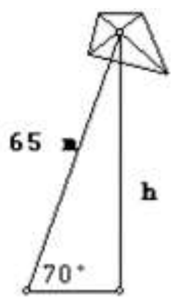
b) $\frac{1 + \frac{sin^2 x}{cos^2 x}}{sec^3 x}$

11.3 Solutions of Right Triangles and Applications

Solution of a right triangle means applying available methods or rules to find any missing part of a right triangle with a minimum of two parts given. These mathematical rules include the Pythagoras Theorem, the six trigonometric functions, the trigonometric identities, and some angle summation postulates.

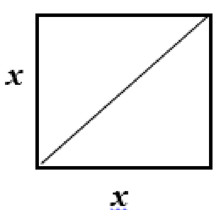
Class Activity 11.3

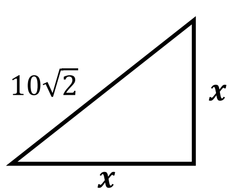
1) Yahya is flying a kite whose string makes an angle of 70° angle with the ground. The kite string is 65 meters long. How far is the kite above the ground? (Round off your answer to 2 decimals).



Solution:

2) What is the measure of each side of a square if its diagonal is $10\sqrt{2}\ cm$?





Solution:

3) Given a right triangle ACB with angle $C = 90°$ and $\cos A = \frac{15}{17}$, do the following:

a) Draw the triangle and labels the parts.

b) Find $\sin A$

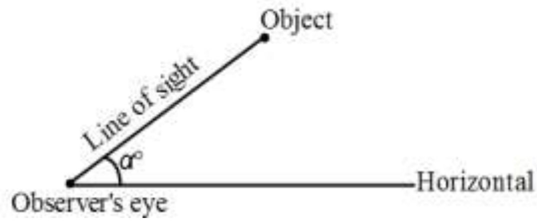
c) Find $\tan A$

d) Find $\sec A$

11.4 Applications Involving Angle of Elevation and Angle of Depression

11.4.1 Angle of Elevation

The **angle of elevation** of an object as seen by an observer is the angle between the horizontal line and the line of sight.

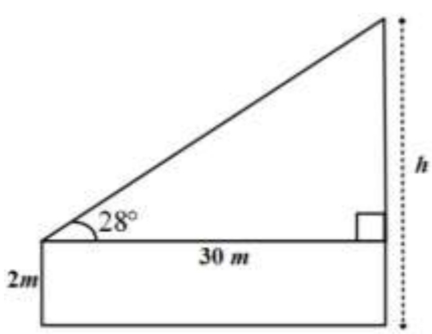


The angle of elevation of the object from the observer is $\alpha°$

Example: A man who is **2*m*** tall stands on horizontal ground **30*m*** from a tree. The angle of elevation of the top of the tree from his eyes is 28˚. Estimate the height of the tree. (Give your answer to 2 decimals).

Solution:

Let the height of the tree be ***h***. Below is the sketch or diagram to represent the situation.



$$\tan 28° = \frac{h-2}{30}$$

$$h - 2 = 30 \tan 28°$$

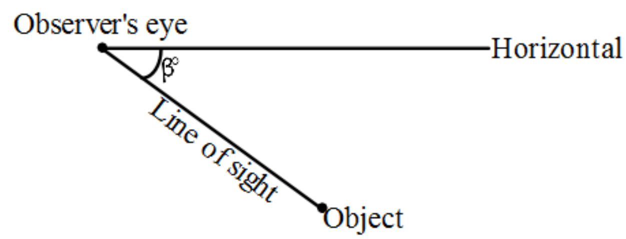
$$h = 30 \tan 28° + 2$$

$$h = 17.95\text{m}$$

The height of the tree is approximately **17.95 *m***.

11.4.2 Angle of Depression

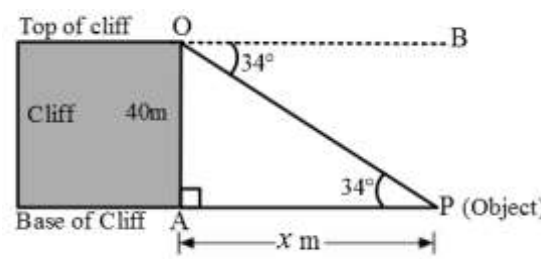
If the object is below the level of the observer, then the angle between the horizontal and the observer's line of sight is called the **angle of depression**.



The angle of depression of the object from the observer is $\beta°$

Example: From the top of a vertical cliff 40 meters high, the angle of depression of an object on the same ground with the base of the cliff is 34°. How far is the object from the base of the cliff? (Give your answer to 2 decimals).

Solution: Let $x =$ the distance of the object from the base of the cliff.



Angle of depression $= \angle BOP = 34°$

$\angle BOP = \angle APO = 34°$

From the triangle APO, we have:

$$\tan 34° = \frac{40}{x}, \quad then \ \ x \tan 34° = 40$$

$$x = \frac{40}{\tan 34°}, \qquad x = 59.30\ m$$

<u>Class Activity 11.4</u>

Answer the following questions/problems using solutions of right triangles.

1. Ahmed wants to measure the height of a tree. He stands exactly 100 feet from the base of the tree and looks up. The angle from the ground to the top of the tree is 33°. How tall is the tree?
   (Round off your answer to 2 decimal places)
   Solution:

2. Find the distance of a boat from a lighthouse if the lighthouse is 100 metres tall, and the angle of depression is 6°.
   (*Give your answer to 2 decimals*).
   Solution:

Link: Some applications of trigonometry

Worksheet 11(a)

Circle the correct answer in the following questions.

1) If $\sin x = \frac{5}{13}$, then $\csc x = \dots$

(a) $\frac{8}{13}$	(b) $\frac{12}{13}$	(c) $\frac{13}{5}$

2) The value of $tan\frac{5\pi}{6}$ is …

(a) $-\frac{1}{\sqrt{3}}$	(b) $-\sqrt{3}$	(c) $\frac{1}{3}$

3) $csc^2y - 1$ is equal to …

(a) sin^2y	(b) tan^2y	(c) cot^2y

4) In a right triangle, $\cos\theta = \frac{4}{5}$, then the value of $\tan\theta$ is …

(a) $\frac{4}{3}$	(b) $\frac{3}{4}$	(c) $\frac{3}{5}$

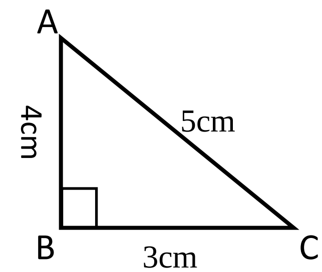
5) The simplest form of $\frac{sin^2\theta \cdot \csc\theta}{\cos\theta}$ is …

(a) $\sin\theta$	(b) $\cos\theta$	(c) $\tan\theta$

6) In a right triangle ABC, $sin\ A = \frac{\sqrt{3}}{2}$. The measure of angle A is …

(a) 60°	(b) 90°	(c) 30°

7) In the right triangle below, $tan\ C = \dots$



(a) $\frac{3}{4}$	(b) $\frac{4}{3}$	(c) $\frac{3}{5}$

8) The example of Pythagorean triplet is …

(a) (2, 3, 4)	(b) (13, 5, 12)	(c) (6, 8, 7)

9)A right triangle ACB has $\angle C = 90°$. Angle ***B*** is 10° more than three times the measure of angle ***A***. What is the measure of angle ***B***?

(a) 70°	(b) 20°	(c) 50°

10)The sides of an equilateral triangle are 16 cm each. What is the height or altitude of this triangle?

(a) 8	(b) $16\sqrt{3}$	(c) $8\sqrt{3}$

Worksheet 11(b)

1. A rope 15 meters long is hooked with one end on a vertical pole. The lower end of the rope is pegged on the ground 10 meters away from the pole. The angle the rope makes with the ground is …

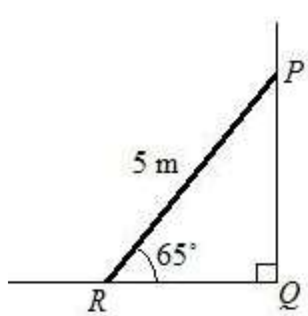
   (a) 48.2°

   (b) 24.8°

   (c) 40°

2. Rashid and Yahya started from the same point but walked to different directions for 1 hour. Rashid reached a distance of 3 km going to the north and Yahya finished 4 km going east. How far from each other were they at the end of 1 hour?

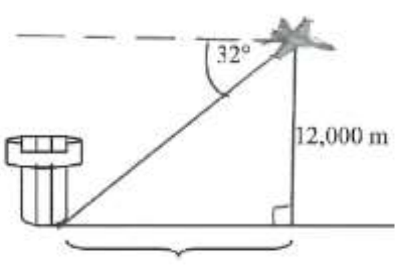
3. A ladder, 5 meters long is leaning against the wall so that its lower end forms 65° with the ground. (Round off your answer to 2 decimals).
   i) How high up the wall the ladder reaches?
   ii) How far is the foot of the ladder from the wall?
   iii) What angle the ladder makes with the wall?



Solutions:

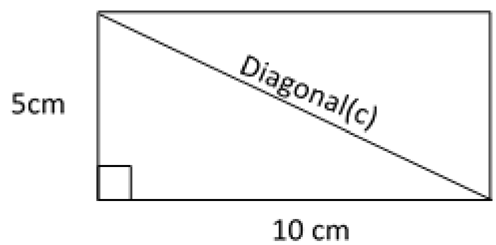
4. Find the length of the shadow cast by a 10 *foot* lamp post when the **angle of elevation** of the sun is 58º. (Round off your answer to 2 decimals).

5. A plane is flying at an altitude of 12 000m. From the pilot, the angle of depression to the base of airport tower is 32^0. How far is the tower from a point directly beneath the plane?

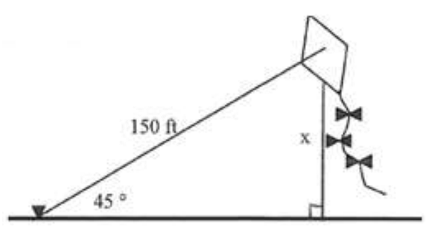


6. A rectangle has sides of lengths 5cm and 10cm. How long is the diagonal of the rectangle?

   (Round off your answer to 2 decimals).
   Solution:



7. A kite with a string 150 feet long makes an angle of 45^0 with the ground. Assuming the string is straight, how high is the kite? (Round off your answer to 2 decimals).



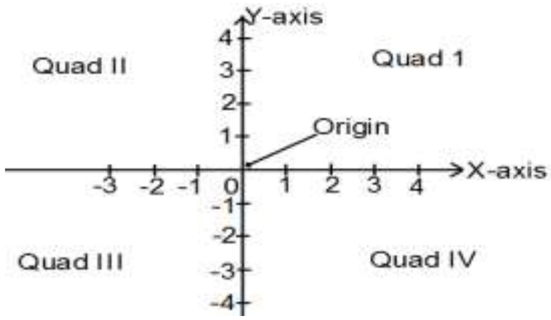
8. A man stands at a point 20 meters away from a building. His eye level is 1.7 meters above the ground. The angle of elevation of the building is 65^0 from this man. How tall is the building?

9. A lifeguard is seated at a lifeguard stand 12 feet above the ground. He is looking down to a swimmer and his line of sight to the swimmer makes an angle of depression of 40^0. How far is the swimmer from the base of the lifeguard stand?

(Unit-12) Coordinate Plane Geometry

12.1 The Rectangular Coordinate System

The Rectangular Coordinate System is made up of two number lines:

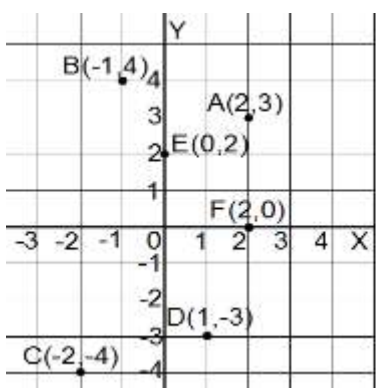


1. The horizontal number line is the X- axis.
2. The vertical number line is the Y- axis.

The origin is where the two lines intersect. This is where both number lines are 0.
The rectangular coordinate plane is divided into four quadrants which are marked as **Quad I** to **Quad IV**.

Each point on the graph is associated with an ordered pair (x, y). To plot the point (x, y), we count first the *x* units from the origin, then followed by *y* units. We call the *x*-coordinate as **abscissa** and the *y*-coordinate as **ordinate**.

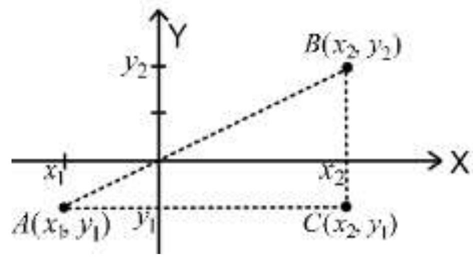
Example: Plot the ordered pairs and name the quadrant or axis in which the point lies.
$A(2,3), B(-1,4), C(-2,-4), D(1,-3), E(0,2)$ and $F(2,0)$.



Solution:
$A(2,3)$ lies in Quadrant I,
$B(-1,4)$ lies in Quadrant II,
$C(-2,-4)$ lies in Quadrant III,
$D(1,-3)$ lies in Quadrant IV,
$E(0,2)$ lies on Y-axis, while $F(2,0)$ lies on the X-axis.

12.2 Distance between two points

Distance is a measure of the length between two points.
Using Pythagoras theorem, we can obtain the distance between two points $A(x_1, y_1)$ and $B(x_2, y_2)$



$AB^2 = AC^2 + BC^2$
$\therefore AB = \sqrt{AC^2 + BC^2}$
$AC = x_2 - x_1$
$BC = y_2 - y_1$
$AB = \sqrt{AC^2 + BC^2}$
$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Note that: $(x_2 - x_1)^2 = (x_1 - x_2)^2$

Example: Find the distance between the points $A(3,9)$ and $B(8,-3)$.
Solution:
Distance $(d) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{(8-3)^2 + (-3-9)^2}$
$= \sqrt{25 + 144} = \sqrt{169} = 13$

Link: Distance between two points

12.3 Midpoint of two points

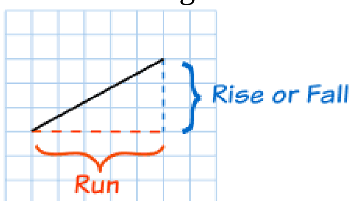
The midpoint of two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$
Example: Find the midpoint of the points $A(3,7)$ and $B(5,-3)$.

Solution: $Midpoint = \left(\frac{3+5}{2}, \frac{7+(-3)}{2}\right) = (4, 2)$

12.4 Gradient or Slope of a line

Gradient (m) describes the slope or steepness of the line joining two points.

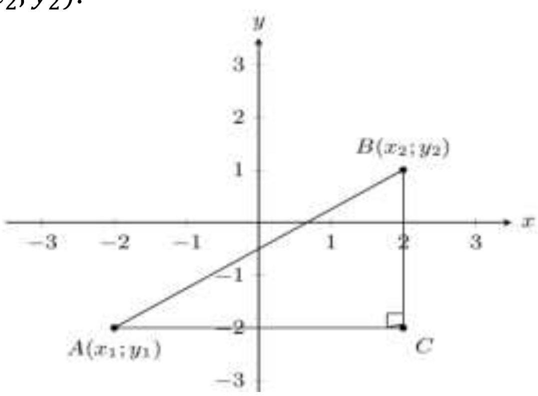
The gradient or slope of a line is determined by the ratio of vertical change to horizontal change.



We can also describe gradient as the ratio of rise or fall to run.

Gradient or slope $(m) = \frac{\text{rise or fall}}{\text{run}}$

Lets consider any two points $A(x_1, y_1)$ and $B(x_2, y_2)$.



Therefore, gradient is determined using the following formula:

Gradient $(m) = \frac{\text{change in } y}{\text{change in } x}$

$$= \frac{y_2 - y_1}{x_2 - x_1} \text{ or } \frac{y_1 - y_2}{x_1 - x_2}$$

Note: remember to be consistent: $m \neq \frac{y_1 - y_2}{x_2 - x_1}$

Example: Find the gradient of the line joining the points $P(1,3)$ and $Q(-2,4)$.

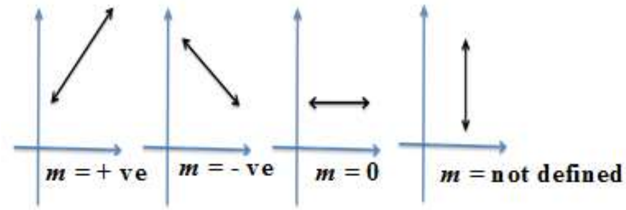
Solution: Gradient, $m = \frac{y_2 - y_1}{x_2 - x_1}$

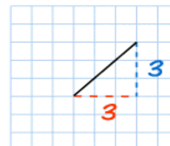
$$m = \frac{4-3}{-2-1} \text{ or } \frac{3-4}{1-(-2)}$$

$$\therefore m = -\frac{\mathbf{1}}{\mathbf{3}}$$

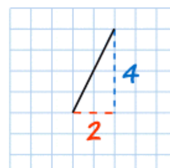
Geometrical interpretation of slope'm'

The diagrams below shows geometrical interpretations of slope/gradient of line.

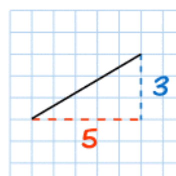




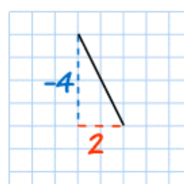
The gradient of this line is $= \frac{3}{3} = 1$
Slope $m = 1$



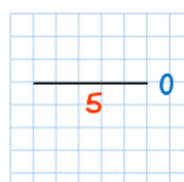
The gradient of this line is $= \frac{4}{2} = 2$
The line is steeper so the gradient is larger



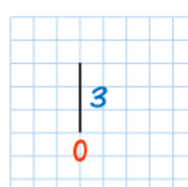
The gradient of this line is $= \frac{3}{5} = 0.6$
The line is less steep so the gradient is smaller



The gradient of this line is $= \frac{-4}{2} = -2$
The line goes down so it has a negative gradient



The gradient of this line is $= \frac{0}{5} = 0$.The horizontal line has a gradient zero.



The gradient of this line is $= \frac{3}{0} =$ undefined. The vertical line's gradient is undefined.

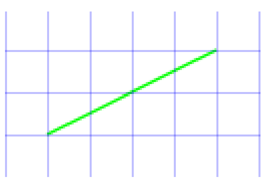
Class Activity 12.1

(1) Find the distance between the points $A(-1, 1)$ *and* $B(3, 4)$.

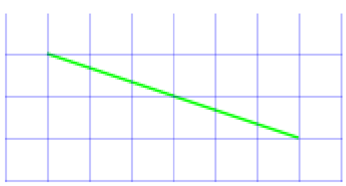
(2) Find the midpoint of the line segment joining the points $A(-4, 1)$ and $B(2, -3)$.

(3) What is the gradient or slope of the line passing through the points $(-3,4)$ and $(3, -4)$?

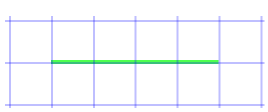
(4) What is the gradient (slope) of this line?



(5) What is the gradient (slope) of this line?



(6) What is the gradient (slope) of this line?



(7) Find the midpoint of the points $A(3,7)$ and $B(5, -3)$.

12.4 Equation of a straight line

The **standard linear equation** in two variables $\boldsymbol{Ax + By = C}$, where A, B and C are constants (with A and B both not zero) and *x* and *y* are variables, is known as the general equation of a straight line.

Example: $y - 2x = 3$.

12.4.1 Drawing the graph of the straight line function based on its equation

X- intercept: The X-intercept of a graph is that value of *x* where the graph crosses the X-axis.

Note:

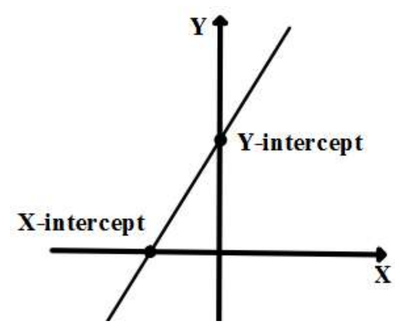
- To find the X-intercept, we have to find the value of x where $y = 0$.
- At *every* point on the X-axis, *y* = 0.

Y-intercept: The Y-intercept is the value of *y* where it crosses the Y-axis.

Note:

- To find the Y-intercept, we have to find the value of y where $x = 0$.
- At *every* point on the Y-axis, $x = 0$.

The *x*- and *y* intercepts of a graph



Example: Find the *x*-intercept and y-intercept and draw the graph of $y = 2x + 10$

Solution: (i) On substituting $y = 0$, we have to solve the equation, $2x + 10 = 0$

$\Rightarrow 2x = -10 \Rightarrow x = -5$

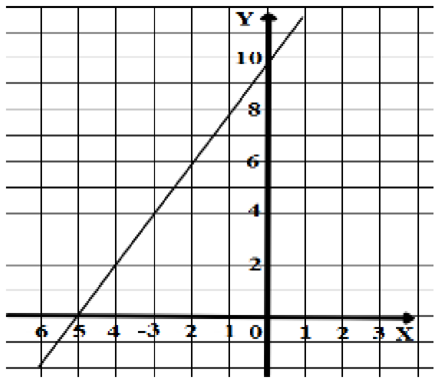
The x-intercept is -5.

(ii) On substituting $x = 0$, we get $y = 10$

The y-intercept is 10.

From (i) and (ii) we get

x	-5	0
y	0	10



Class Activity 12.2

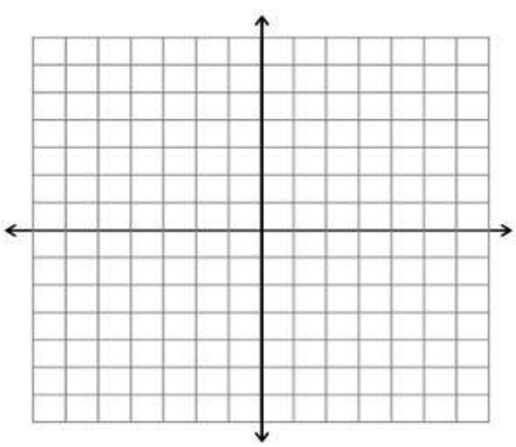
1. Find the x-intercept and y-intercept of the equation: $y = 3x - 12$

   Solution:

2. Complete the table and draw the graph of $y = 2x + 6$

   Solution:

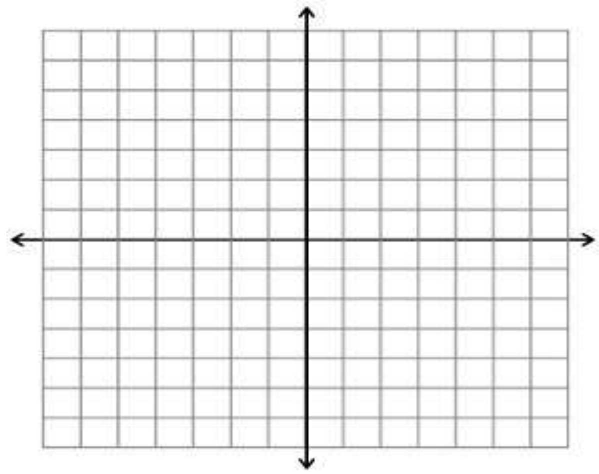
x		
y		



3. Complete the table and draw the graph of $y = -3x + 3$

   Solution:

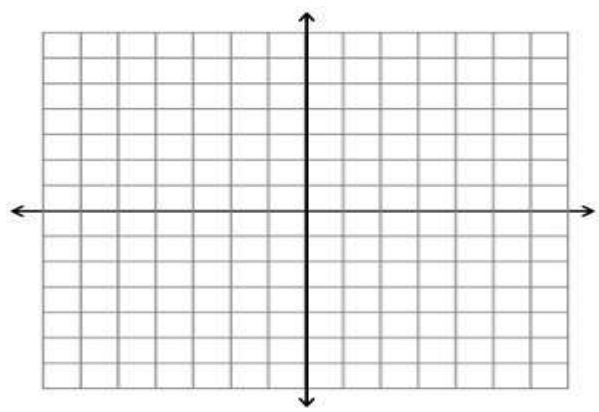
x		
y		



4. Complete the table of values and draw the graph for $y = 2x$

   Solution:

x	0	2
y		



12.4.2 Slope-intercept form

The equation of the line $Ax + By = C$ can be written as $By = -Ax + C$

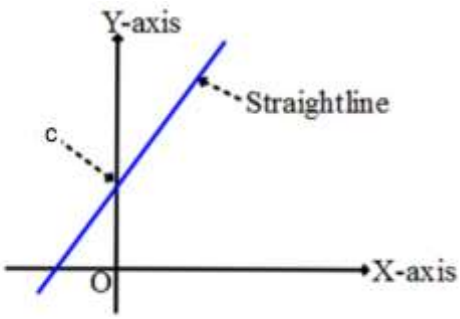
or $y = -\frac{A}{B}x + \frac{C}{B}$

or $y = mx + c$,

where m = $-\frac{A}{B}$ = Slope of the line.

and c = y-intercept = $\frac{C}{B}$.

The equation $\boldsymbol{y = mx + c}$ is known as the slope-intercept form of the equation of a straight-line, where $\boldsymbol{m}$ = gradient/slope
$\boldsymbol{c}$ is the Y-intercept or point where the graph crosses the Y-axis.



Note:

1. The horizontal axis is called the X-axis
2. The vertical axis is called the Y-axis
3. The point O(0,0) is called the origin
4. *x* is the independent variable
5. *y* is the dependent variable
6. Generally the independent variable is plotted along the horizontal axis and the dependent variable along vertical axis.

Example 1:

Write the equation of the line with slope, $m = 2$ and Y-intercept, $c = 1$

Solution:
Given m = slope = 2 and c = Y-intercept = 1
∴ substituting these values in the equation of straight line: $\boldsymbol{y = mx + c}$, we get $\boldsymbol{y = 2x + 1}$

Example 2:

Find the slope and the Y-intercept of the line $-12x + 3y = 7$.

Solution:
The given equation can be written as
$3y = 12x + 7$ **or** $y = \frac{12}{3}x + \frac{7}{3}$
which gives $y = 4x + \frac{7}{3}$

Comparing with $\boldsymbol{y = mx + c}$,
We get m = slope = 4 and c = Y-intercept = $\frac{7}{3}$

Example 3:

Find the equation of the line in the form:

$y = mx + c$ which passes through the point $(-4, 2)$ and the intercept on the Y-axis is 6.

Solution:
Given that intercept on Y-axis = $c = 6$, so we have $y = mx + 6 \ldots(1)$
(from $y = mx + c$) passes through $(-4, 2)$

Hence: $2 = m(-4) + 6$ or $4m = 4 \rightarrow m = 1$

∴ from (1) $y = 1x + 6$

and $\boldsymbol{y = x + 6}$ is the required equation of the line.

Example 4: If the straight line $y = mx + c$ passes though the points (1, 1) and (2, 4) then find the values of m and c.

Solution: The line $y = mx + c$, passes through $(1, 1)$ so $1 = m(1) + c$ or

$$1 = m + c \text{ ... } (1)$$

The line also passes through $(2, 4)$ so

$$4 = 2m + c \ldots (2)$$

Subtracting (1) from (2) we get

$$3 = m \quad \text{or} \quad m = 3$$

Substituting the value of m in $y = mx + c$, we get $1 = 3 + c$ or $c = -2$

Alternative method for obtaining $'m'$:

$m = \frac{y_2-y_1}{x_2-x_1} = \frac{4-1}{2-1} = 3$

12.4.3 Point-Slope Form

$$(\boldsymbol{y} - \boldsymbol{y_1}) = \boldsymbol{m}(\boldsymbol{x} - \boldsymbol{x_1}) \text{ ,}$$

For the same **Example 4,** given earlier

Using one of the points $(1,1)$ and $m = 3$

We get $(y - 1) = 3(x - 1)$

$y = 3(x - 1) + 1 \rightarrow y = 3x - 3 + 1$

$\therefore$ The equation of line is $\boldsymbol{y = 3x - 2}$

Link: Equation of a straight line

Class Activity 12.3

1. Write the equation of the line when $m = -3$ and $c = 5$.

2. Find m and c from the equation: $6x + 3y = 9$

3. Find equation of line if $c = 3$ and the line passes through $(1, -1)$

4. Find the equation of the line if it passes through $(2,3)$ and $(0, 0)$.

12.5 Parallel and Perpendicular lines

Remember:
Equation of a straight line is $\boldsymbol{y = mx + c}$
or $\boldsymbol{(y - y_1) = m(x - x_1)}$

Let m_1 be the slope of the line L_1 and
m_2 be the slope of the line L_2

Case-1: If L_1 is parallel to L_2 then $m_1 = m_2$
Parallel lines have the same gradient

Case-2: If L_1 perpendicular to L_2 then
$m_1 \times m_2 = -1$

Example 1:
Find the equation of a line passing through the point (4, –7) parallel to the line $4x + 6y = 9$.

Solution:

Step-1: Find the slope of L_1: $4x + 6y = 9$
$\Rightarrow y = \frac{-2x}{3} + \frac{3}{2}$, so $m_1 = -\frac{2}{3}$.
$m_1 = m_2 = -\frac{2}{3}$ because the two lines are parallel.

Step-2: Find the equation of L_2 by using the point-slope form: $(y - y_1) = m(x - x_1)$ where m = m_2 .
Hence the equation of a line passing the given point $(x_1, y_1) = (4, -7)$ is

$\Rightarrow (y + 7) = -\frac{2}{3}(x - 4)$
$\Rightarrow 3y + 21 = -2x + 8$

$\Rightarrow 3y + 2x = -13$
Therefore the required equation is
$$L_2:\ y = -\frac{2}{3}x - \frac{13}{3}$$

Link: Equations of parallel and perpendicular lines

Example 2: A line passes through $(3, 11)$ and is **perpendicular** to the line: $y = 3x - 4$.
Find the equation of the line.

Solution:

The gradient of L_1: $y = 3x - 4$ is $m_1 = 3$, so the gradient of the perpendicular line is

$m_2 = -\frac{1}{3}$ because $m_1 \times m_2 = -1$.

Using the point-slope form:

$(y - y_1) = m(x - x_1)$, with $m_2 = -\frac{1}{3}$, and the point $(x_1, y_1) = (3,11)$.

$y - 11 = -\frac{1}{3}(x - 3)$,

$y = -\frac{1}{3}x + 1 + 11$,

$y = -\frac{1}{3}x + 12$, is the equation of L_2.

Class Activity 12.5

1. A line passes through $(3, 11)$ and is **parallel** to the line $y = 3x - 4$. Find the equation of the line.

2. Find the equation of a line passing through the point $(-3,8)$ and is **perpendicular** to the line $2x - 7y = -11$.

Worksheet 12

1) The slope of the **X**-axis is _______.
   (a) 0 (b) 1 (c) Not defined

2) The slope of Y- axis is _______.
   (a) 0 (b) 1 (c) Not defined

3) The slope of the line joining the points $(2,-3)$ and $(3,-2)$ is _______.
   (a) 0 (b) 1 (c) -1

4) The slope of the line $4x+2y=5$ is…
   (a) 4 (b) 2 (c) -2

5) The y-intercept of the line $4x+3y=6$ is
   (a) 4 (b) 2 (c) -2

6) What is the length of the radius of a circle if one of its diameters has endpoints at (-3, 4) and (6, -8)?

   (a) 15 (b) 12 (c) 7.5

7) What is the equation of the line that has X-intercept at $\boldsymbol{x = 5}$ and Y-intercept at $\boldsymbol{y = -2}$?
   (a) $2x - 5y = 10$
   (b) $2x + 5y = 10$
   (c) $-2x + 5y = 10$

8) Find the distance between the two points $(-5,-3)$ and $(4, 2)$
   (*Give answer to 1 decimal place*)

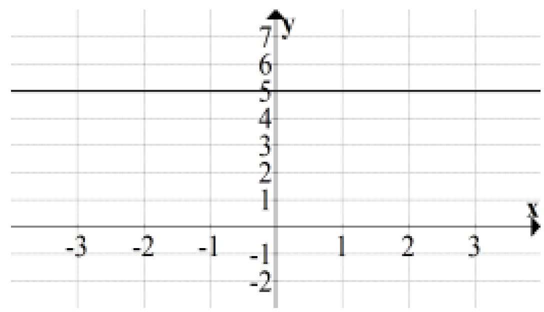
9) Find the equation of the line parallel to $2y - x = 6$ and passing through the point (2, 0).

10) Find the equation of the line perpendicular to $y = \frac{1}{3}x - 4$ and passing through the point (4, -5).

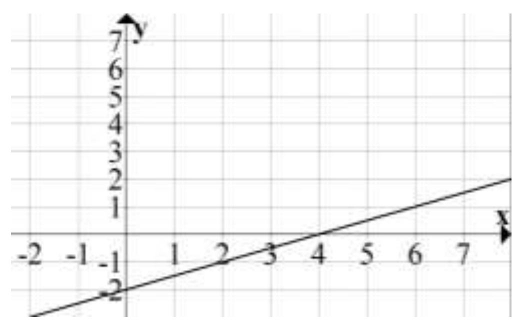
11) For what value of a is the line $y = ax$, perpendicular to the line $\frac{2}{3}y - 6x = 3$?

12) Write down the equation of the line from the following graphs

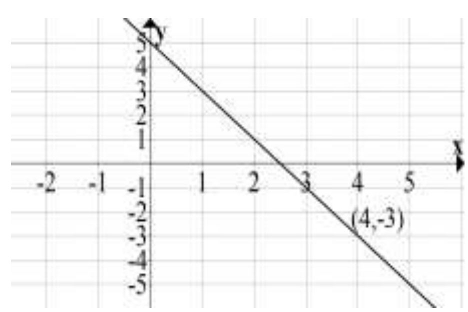
i)



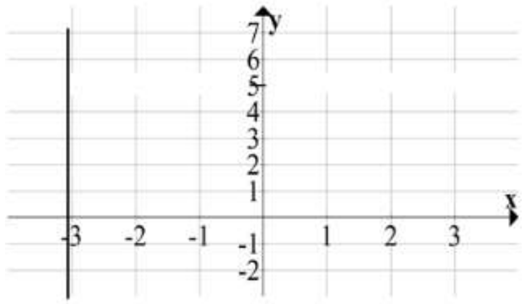
ii)



iii)



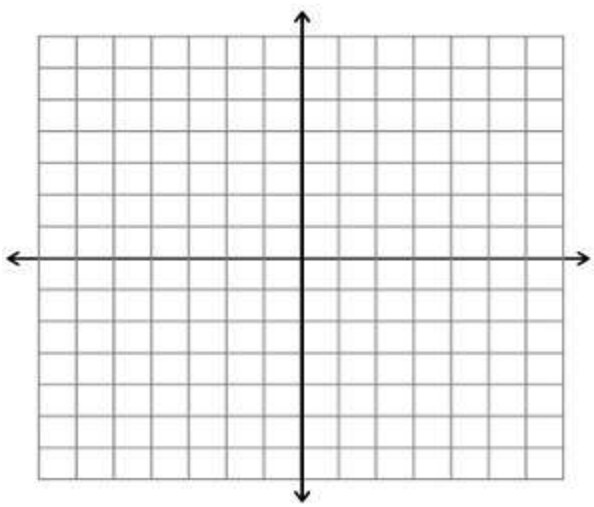
iv)



13) Complete the table and draw the graph of $y + x = 2$

Solution:

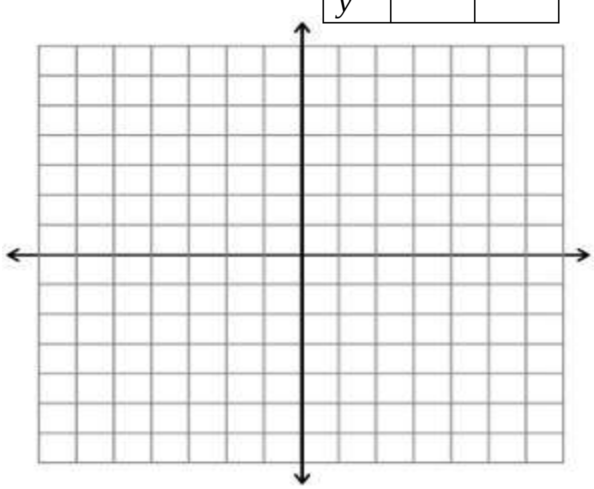
x		
y		



14) Complete the table and draw the graph of $y = -\frac{1}{2}x + 2$

Solution:

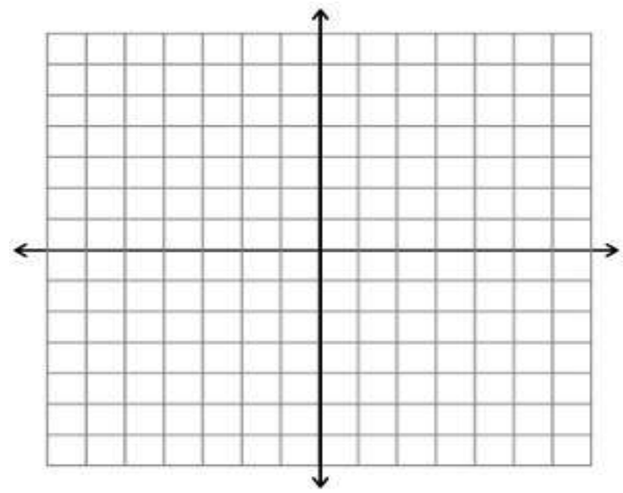
x		
y		



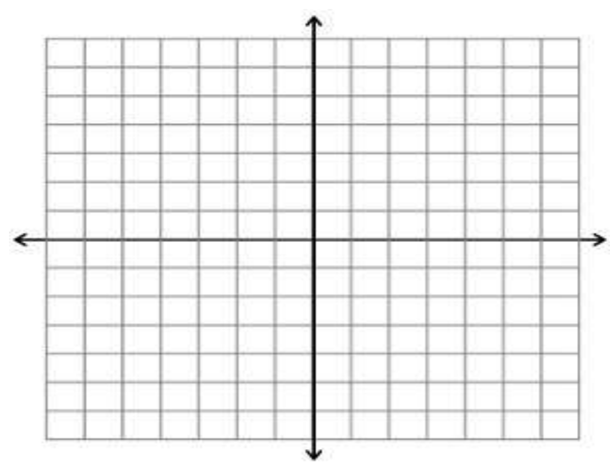
15) Complete the table of values and draw the graph for $y = 3x$

Solution:

x		
y		



16) Draw the graph of $y = -2$



(Unit-13) Circle Coordinate Geometry and Symmetry

13.1 Centre and radius of circle, Tangent lines

Definition: A **circle** is the set of all points in a plane equidistant from a fixed point. The fixed point is called the **centre** and the fixed distance is called the **radius**.

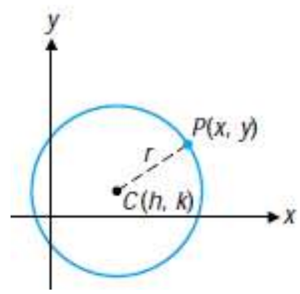
As stated in its definition, the equation of the circle can be found by using the distance formula. Suppose that the point $C(h, k)$ is the centre and the circle has radius r, where $r > 0$. Let $P(x, y)$ represents any point on the circle as shown below.

$$\sqrt{(x-h)^2+(y-k)^2} = r$$

(By distance formula)

$$(x-h)^2+(y-k)^2 = r^2$$

(Squaring both sides)



This is called **centre-radius form** of the equation of the circle. As a special case, a circle with radius r and centre (0, 0) has the following equation.

$$x^2 + y^2 = r^2$$

General form of equation of circle:

$$x^2 + y^2 + cx + dy + e = 0$$

where c, d and e are some real numbers.

This general form of the equation can be written in the form $(\boldsymbol{x}-\boldsymbol{h})^2+(\boldsymbol{y}-\boldsymbol{k})^2=\boldsymbol{r}^2$ by completing the squares.

Hence, the standard equation of a circle is

$$(\boldsymbol{x}-\boldsymbol{h})^2+(\boldsymbol{y}-\boldsymbol{k})^2=\boldsymbol{r}^2$$

where $(\boldsymbol{h},\boldsymbol{k})$, are the coordinates of the centre and the radius is $\boldsymbol{r}$.

Example 1: Find the equation of the circle with radius 4 and centre at $A(-3,6)$ and draw the graph.

Solution:

Here centre $(h, k) = (-3,6)$ and radius: $r = 4$

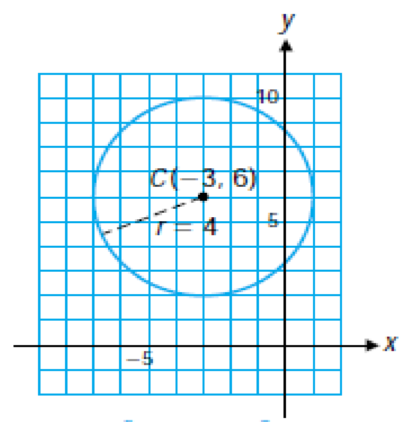
The equation of the circle is given by

$$(x-h)^2+(y-k)^2=r^2$$

$$\therefore (x-(-3))^2+(y-6)^2=4^2$$

$$(x+3)^2+(y-6)^2=16$$

To graph the circle, locate the centre$(-3,6)$ and draw a circle of radius 4



$(x+3)^2+(y-6)^2=16$

Example 2: Find the radius and centre if the equation of the circle is: $x^2+y^2=16$, and draw the graph.

Solution:

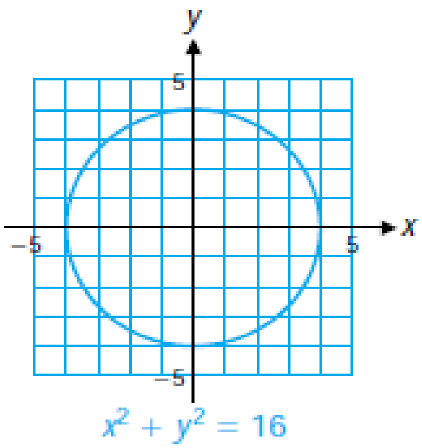
The equation of the circle is

$$(\boldsymbol{x}-\boldsymbol{h})^2+(\boldsymbol{y}-\boldsymbol{k})^2=\boldsymbol{r}^2$$

$$\therefore (x-0)^2+(y-0)^2=4^2$$

Here centre $(h, k) = (0,0)$ and radius: $r = 4$

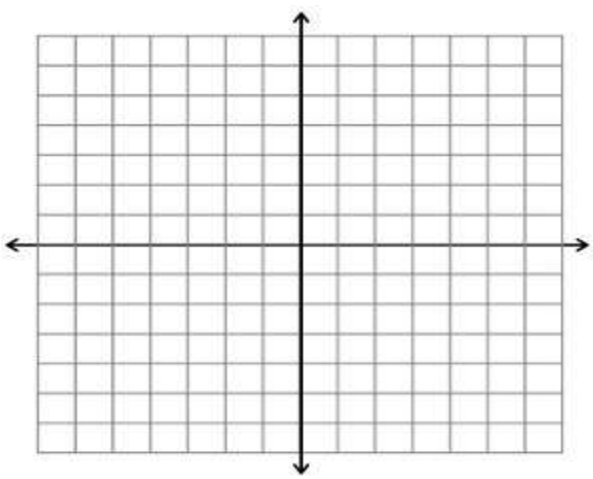
To graph the circle, locate the centre $(0,0)$ and draw a circle of radius 4



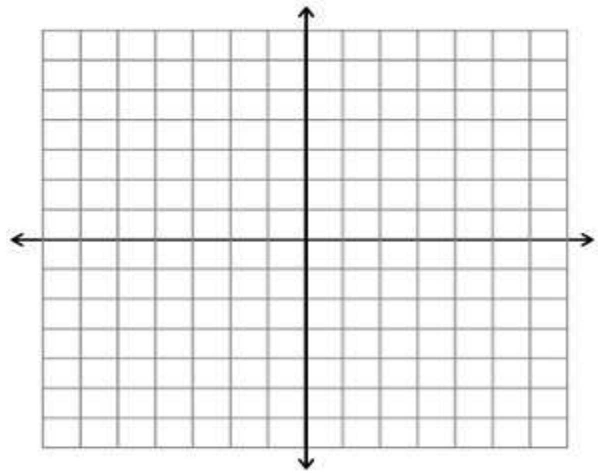
Link: Equation of Circle

Class Activity 13.1.1

1) Find the equation of the circle with radius 3 and centre at $(0,0)$ and draw the graph

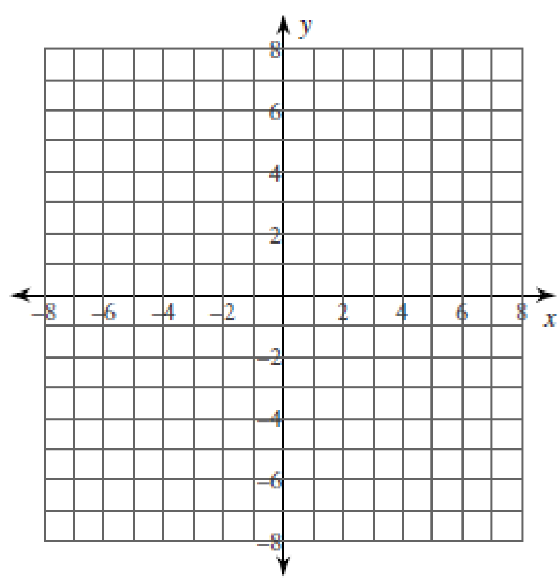


2) Find the equation of the circle with radius 3 and centre at $A(3,-2)$ and draw the graph.

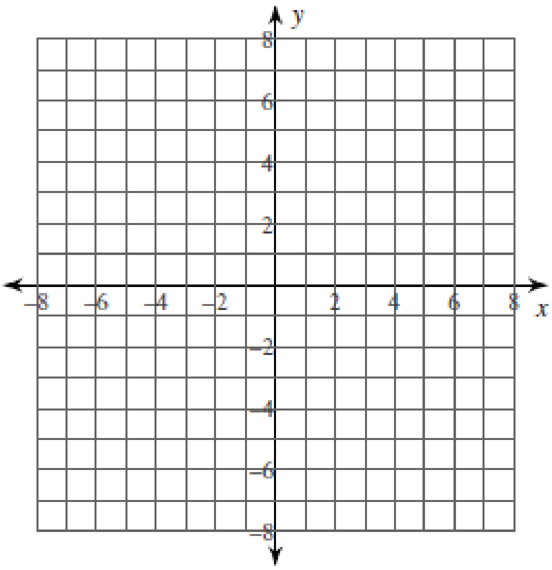


3. Identify the centre and radius from each equation, then sketch the graph.

(i) $(x-1)^2+(y+3)^2=4$



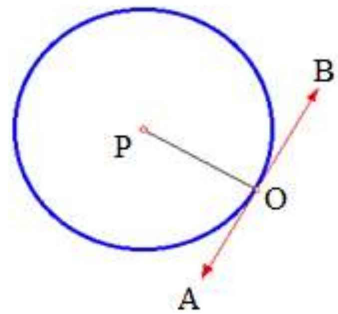
(ii) $(\boldsymbol{x}+\mathbf{2})^{\mathbf{2}}+(\boldsymbol{y}-\mathbf{1})^{\mathbf{2}}=\mathbf{16}$



4) Q (-4,7) is a point on a circle with centre at C(-1,3). Find the equation of the circle.

5) The diameter of a circle has two endpoints: P (-6,0) and Q (0,8). Find the equation of the circle.

Tangent line: Is a line that touches the circle external at one point only.



Note:

- OP is the radius of the circle
- AB is the tangent line to the circle
- The tangent AB is perpendicular to OP, the radius of the circle.

Example: Find the equation of the circle with tangent $x=4$ and centre at A(2,6).

Solution: In this case, the radius is the distance between the x coordinate of the centre and the value of x at the point of tangency.

$$r=|4-2|$$

$$r=2$$

The equation of the circle is

$$(\boldsymbol{x}-\boldsymbol{h})^{\mathbf{2}}+(\boldsymbol{y}-\boldsymbol{k})^{\mathbf{2}}=\boldsymbol{r}^{\mathbf{2}}$$

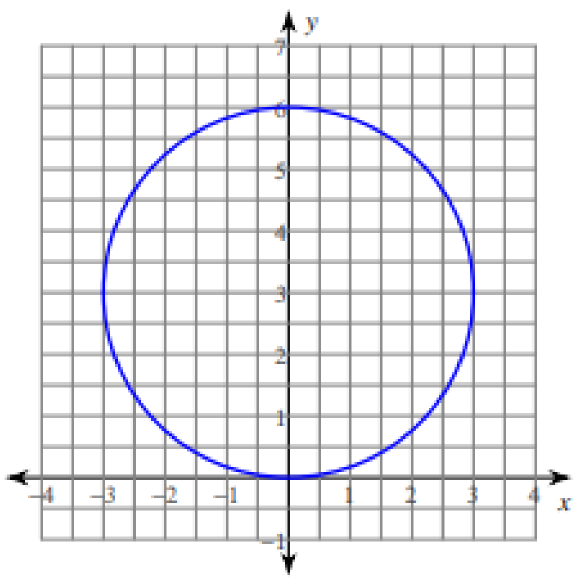
$$(x-2)^2+(y-6)^2=4$$

Class Activity 13.1.2

1. Find the equation of the circle with centre (5, 2) and with a tangent $y = -2$
(Hint: radius r is the distance between the centre and the point of tangency)

2. Find the equation of the circle with centre (-2, 12) and with a tangent $x = -5$.

4) From the circle represented in the graph below:



i) State the coordinates of the centre and the value of the radius.

ii) Write down the equation of the circle.

iii) Write down the equations of the tangents to the the circle at the points shown in parts (a) and (b).

(a) (0, 6)

(b) (-3,3)

13.2 Symmetry of Graphs

Meaning of Symmetry

Symmetry comes from a Greek word meaning 'to measure together' and is widely used in the study of geometry.

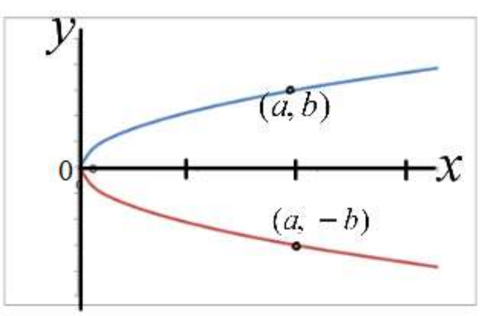
Mathematically, **symmetry** means that one shape becomes exactly like another when you move it in some way: turn, flip or slide. For two objects to be symmetrical, they must be the same size and shape, with one object having a different orientation from the first. There can also be symmetry in one object, such as a face. If you draw a line of symmetry down the center of **your face**, you can see that the left side is a mirror image of the right side. Not all objects have symmetry; if an object is not symmetrical, it is called **asymmetric**.

In this section we are going to study three types of symmetry. However, we're going to take a more general view of symmetry in this section.

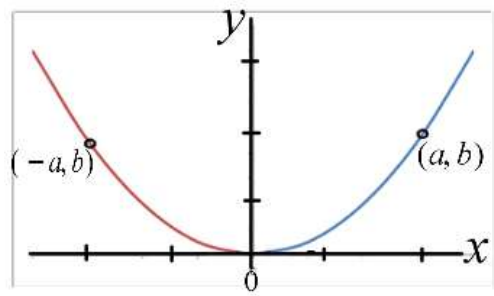
Symmetry can be useful in graphing an equation since it says that if we know one portion of the graph then we will also know the remaining (and symmetric) portion of the graph as well.

Let us try to understand the concept of the symmetry through the following graphs

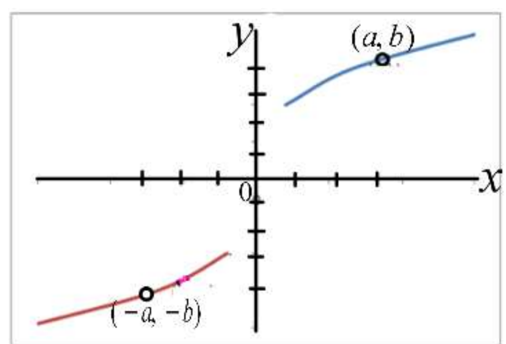
1. A graph is said to be **symmetric about *x*-axis** if whenever (a, b) is on the graph then so is (a, -b). Here is a sketch of graph that is symmetric about *x*-axis.



2. A graph is said to be **symmetric about y-axis** if whenever (a, b) is on the graph then so is (-a, b). Here is a sketch of graph that is symmetric about y-axis.



3. A graph is said to be **symmetric about the origin** if whenever (a, b) is on the graph then so is (-a, -b). Here is a sketch of graph that is symmetric about the origin.

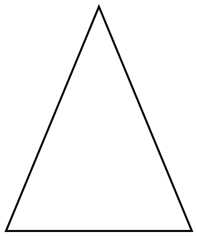


Link: Lines of Symmetry

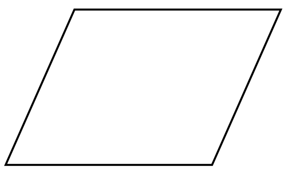
Class Activity 13.2

1) How many lines of symmetry are there in the following shapes?

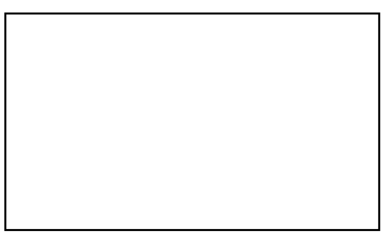
(i)



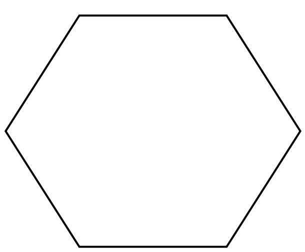
(ii)



(iii)

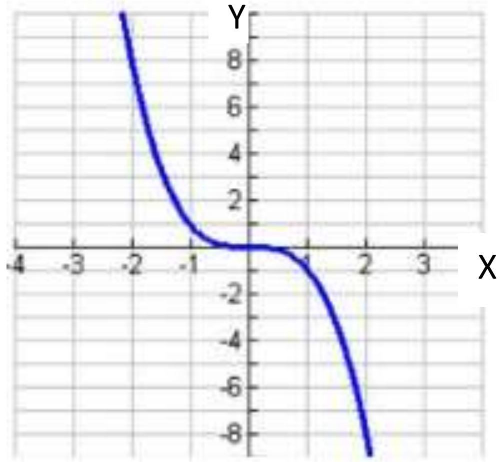


(iv)

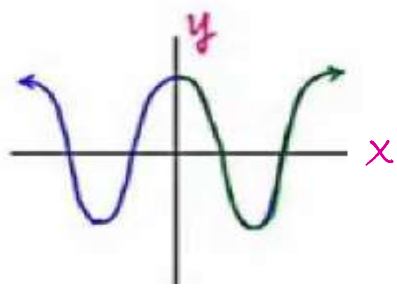


2) Identify the type of symmetry shown in the diagrams below

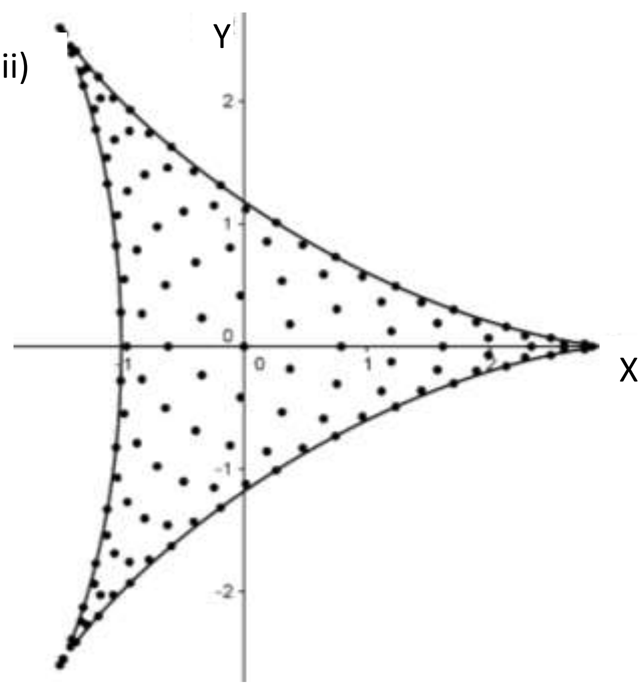
i)



ii)



iii)



Work Sheet 13 (a)

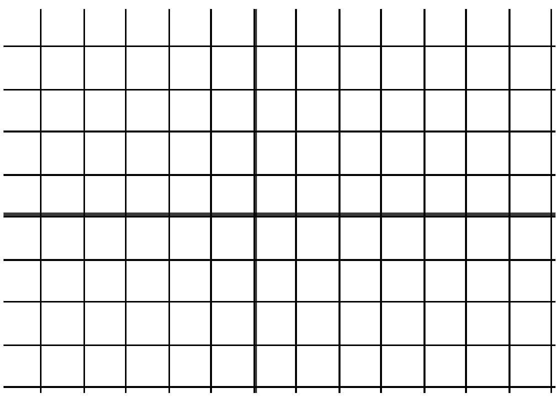
1) In the following problems write the equation of the circle with the indicated center and radius.

   i) Centre: $(0,0), radius, r = 4$

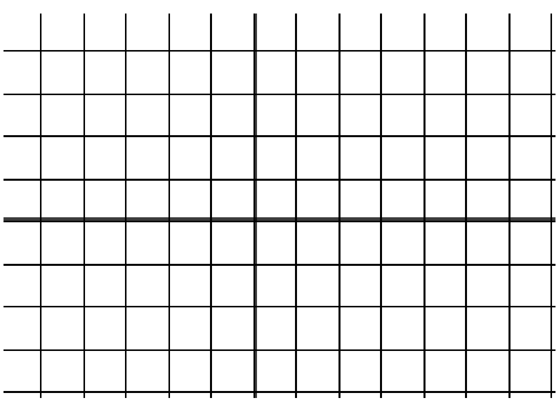
   ii) Centre: $(-4,2), radius, r = 5$

2) In the following problems find the centre and radius of the circle with the given equation. Draw the graph of the equation

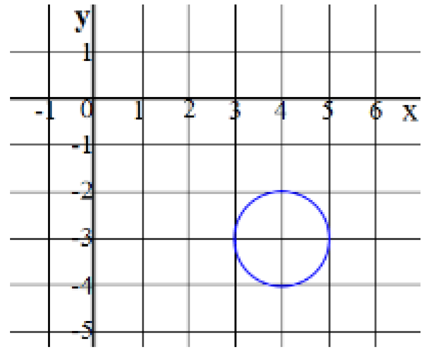
   i) $x^2 + y^2 = 9$



(ii) $(x+4)^2 + (y-2)^2 = 4$



3) From the graph below:

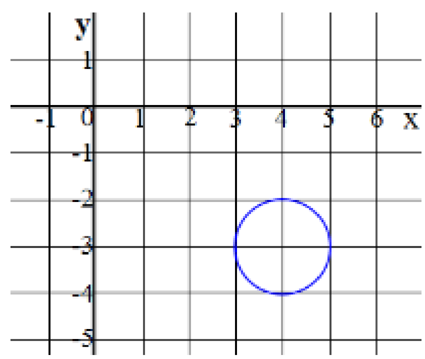


i) State the coordinates of the center of the circle

ii) Find the length of the radius

iii) Write down the equation of the circle

4) Using the graph below write down the equations of the tangents to the circle at the points indicated:



i) (4, -2)

ii) (3, -3)

5) Identify the center and the radius of each circle represented by the equations in the table below.

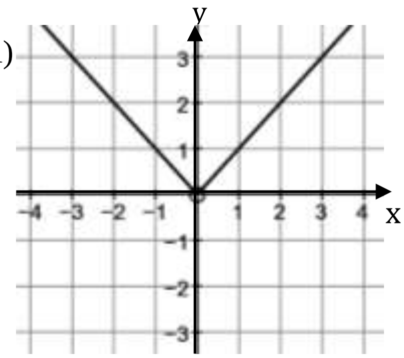
Eq.#	**Standard Graphing Equation**	**Radius**	**Center**
i)	$(x-2)^2+(y-5)^2=144$		
ii)	$x^2+(y-7)^2=625$		
iii)	$(x-15)^2+(y+8)^2=289$		
iv)	$(x+11)^2+(y+6)^2=324$		
v)	$(x+4)^2+(y-9)^2=196$		

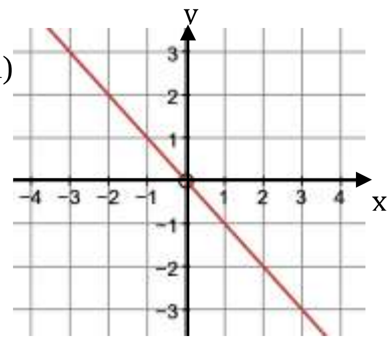
6) Given the centre and radius in the table below, write the standard equation of each circle

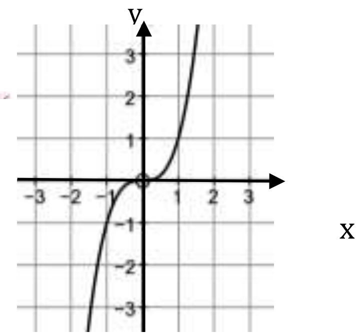
Eq.#	**Centre**	**Radius**	**Standard Graphing Equation**
i)	$(-5,12)$	13	
ii)	$(9,-12)$	15	
iii)	$(-7,0)$	10	
iv)	$(11,14)$	9	
v)	$(-6,-8)$	5	

<u>Worksheet 13 (b)</u>

1. The following graphs all show symmetry about the origin except:

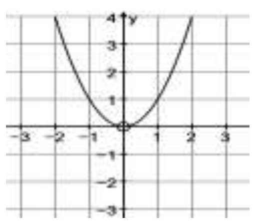
i)


ii)


iii)


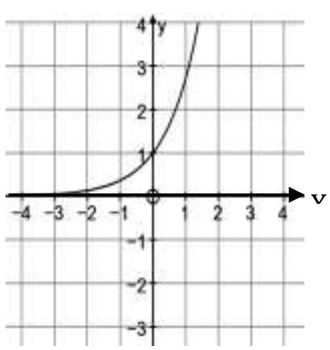
   a) i
   b) ii
   c) iii

2. The following graph shows symmetry about …



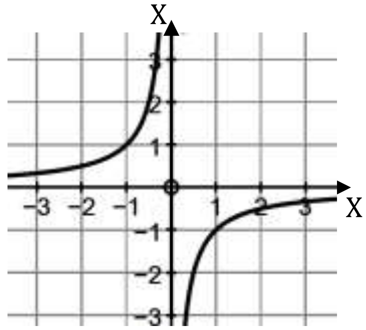
   a) Origin
   b) x -axis
   c) y-axis

3. The graph shown here is …



   a) Asymmetrical
   b) Symmetrical about the y-axis
   c) Symmetrical about the origin

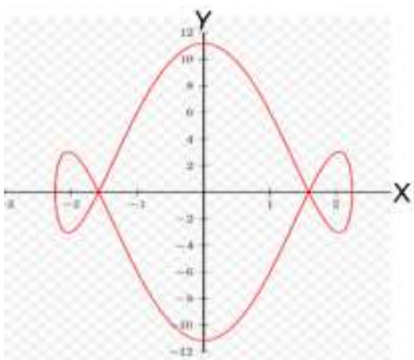
4. The graph shown here is



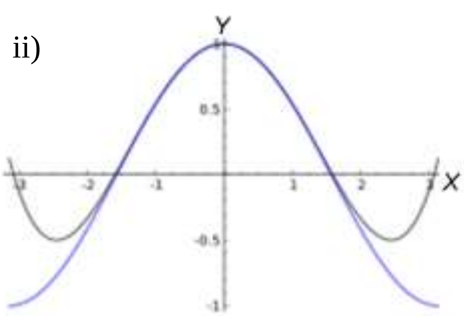
a) Asymmetrical
b) Symmetrical about both origin and x-axis
c) Symmetrical about the origin

5. State what type of symmetry is shown in the following graphs.

i)



ii)



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