MILITARY TECHNOLOGICAL COLLEGE

FOUNDATION PROGRAM DEPARTMENT

GFP EXIT SAMPLE EXAM PAPER (with Answers at the end)

Module name	Pure Mathematics	Module code	MTCG1018
Date		Duration of exam	90 Minutes

STUDENT DETAILS

Student ID.		Signature of Student	
Seat Number			

Instructions:

- Complete the information required on the front page. Use blue / black ink or ball-point pen.
- Students should not keep any helping / study materials with them. Copying, cheating and any kind of malpractice in the examination are strictly prohibited.
- Use of only non- programmable calculators is allowed.
- Answer the questions in the space provided. Extra sheets will not be allowed or provided.
- All necessary steps for solutions must be shown in Section B and Section C, otherwise marks for method will be lost.
- The figures shown, if any, are only for illustration.
- A short list of relevant **Formulas** is attached at the back.
- **Do not open** this question paper until the invigilator has told you to do so.
- **This exam carries 100% of the overall module mark.**

MARK DISTRIBUTION			
Section	No. of Questions × Marks per Question = Total Marks Allocated	1st Marking	2nd Marking
A	20 × 1 = 20 Marks		
B	10 × 2 = 20 Marks		
C	5 × 2 = 10 Marks		
	Total = 50 Marks	/50	/50
Final Marks		/50	

Section-A

Circle the correct answer in the following questions. ☞ (a)

Each question carries one mark. **[Total Marks 20]**

1) The radius of a sector is 5 *cm* and corresponding arc length is 10 *cm*, then the area of the sector is …

a) 25 cm^2	b) 36 cm^2	c) 12 cm^2

2) If the volume of a cube is 27 cm^3, then the side is …

a) 25 *cm*	b) 10 *cm*	c) 3 *cm*

3) … refers to the number of observations chosen randomly in order to investigate the population.

a) Statistics	b) Sample size	c) Data

4) ...is a measure of dispersion.

a) Mean	b) Median	c) Range

5) If a die is rolled 2 times, then the sample size is…

a) 36	b) 18	c) 3

6) … distinct permutations can be made with the letters of the word “TOWN”

a) 24	b) 6	c) 120

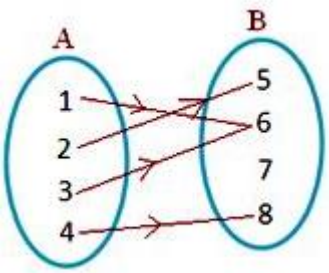
7) If a card is drawn from an ordinary deck of cards, then the probability that it is Red is...

a) $\frac{1}{13}$	b) $\frac{1}{26}$	c) $\frac{1}{2}$

8) If D, E and F are mutually exclusive events and P(D)=0.4, P(E)=0.3 and P(F)=0.2, then $P(F')$ is..

a) 0.6	b) 0.8	c) 0.9

9) In the following figure the codomain is …



a) {5, 6, 7, 8}	b) {1, 2, 3, 4}	c) {5, 6, 8}

10) The ordered pairs… represent a function.

a) {(2, 7), (3, 7), (4, 7), (5, 7)}	b) {(1,-5), (1, 6), (2, 6), (6,-3)}	c) {(1,-5), (4, 1), (-5, 4), (4,-2)}

11) … represents a constant function.

a) $g(x) = 5x$	b) $g(x) = 2x^2 - 3$	c) $g(x) = 7$

12) The number of times a vertical line crosses the graph of a function is…

a) 0	b) 1	c) 2

13) …is an exponential function.

a) $f(x) = 5^x - 1$	b) $f(x) = sinx$	c) $f(x) = 3x^2$

14) From the statements below, … is true.

a) $log_3 6 = 2$	b) $log_4 16 = 2$	c) $log_2 25 = 5$

15) Using properties of logarithms, $log\ (\frac{b^5}{a})$ can be written as …

a) $5logb + 5loga$	b) $loga + 5logb$	c) $5logb - loga$

16) The value of $\lim_{x\to\frac{1}{3}}\left(\frac{9x^2-1}{3x-1}\right)$ is…

a)0	b) 2	c) 3

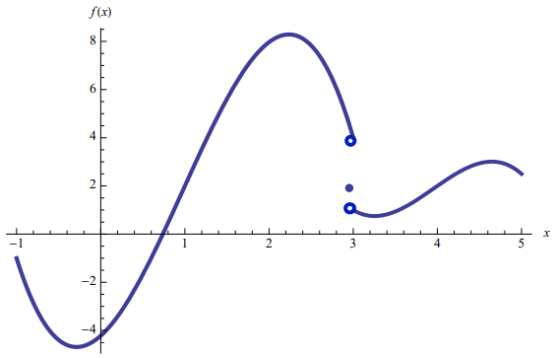
17) If $y = 3x^2 - 5x + 4$, then $\frac{dy}{dx}$ =…

a) $6x - 5$	b) $3x - 5$	c) $6x + 4$

18) If $y = \frac{2}{3x}$, then $\frac{dy}{dx}$ =…

a) $-\frac{1}{3x^2}$	b) $\frac{1}{3x^2}$	c) $-\frac{2}{3x^2}$

19) For the function $f(x)$ whose graph is given below, the correct statement is…



a) $f(3) = \lim_{x\to 3} f(x)$	b) $\lim_{x\to 3^-} f(x) = 4$	c) $\lim_{x\to 3^-} f(x) = \lim_{x\to 3^+} f(x) = 2$

20) The derivative of $xcos3x$ is …

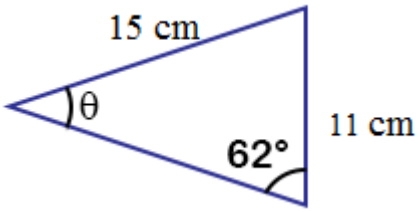
a) $cos3x - 3xsin3x$	b) $3cos3x - 3xsin3x$	c) $cos3x + 3xsin3x$

Section-B

Show your solution step by step in the following questions.

Each question carries two marks. **[Total Marks 20]**

1) In the following triangle find 'θ'. **(Write the answer to 1 decimal place)**



Solution:

2) Height of a cylinder is twice as its diameter and its volume is $864\pi\ cm^3$. What is its radius?

Solution:

3) What is the mode of the following data?
(Write the answer to 1 decimal place)

Marks (out of 50)	**No. students**
1 – 10	5
11 – 20	7
21 – 30	10
31 – 40	8
41 – 50	2

Solution:

4) A random sample of 300 children are classified below according to gender and the level of education attained.

	Male	Female
Nursery	40	45
Primary School	35	45
High School	85	50

If a person is picked at random from this group, what is the probability that the person does not have a high school certificate, given that the person is a female? **(Write the answer to 1 decimal place)**

Solution:

5) If $f(x) = x - 1$ and $g(x) = x^2 - 3x$, find $(g \circ f)(-3)$

Solution:

6) NH4 flue disease, is caused by a virus that multiplies exponentially by cell division as modelled by $V = V_0 e^{1.285\,t}$

Where V is the number of virus present after t hours and V_0 is the number of virus present at $t = 0$. If we start with 12 virus, how many virus will be present in 5 hours?

(Write the answer to 2 Significant figures)

Solution:

7) Find the value of 'x' in the equation: $5^{x+1} = 8$. **(Write the answer to 1 decimal place)**

Solution:

8) Evaluate $\lim_{x\to\infty}\left(\frac{x-1}{2x+3}\right)$.

Solution:

9) Determine the $\lim_{x\to 2} h(x)$ when h is defined as follows:

$$h(x) = \begin{cases} \dfrac{7x+6}{2}, & if\ x < 2 \\ 3x+4, & if\ x \geq 2 \end{cases}$$

Solution:

10) Find the derivative of the function $y = (3x^2 - 2x)^5$

Solution:

Section-C

Show your solution step by step in the following questions.

Each question carries two marks. **[Total Marks 10]**

1) For the function *f*(*x*) defined below, determine the value of b so that $\lim_{x\to 5} f(x)$ exists.

$$f(x) = \begin{cases} 2x - 3 \; if \; x < 5 \\ \frac{2}{3}x + b \; if \; x \geq 5 \end{cases}$$

Solution:

2) When 9 m^3 of water is removed from a cylindrical tank, the level of water in the tank goes down by 2.5 *m*. What is the radius of the tank? **(Write the answer to 1 decimal place)**

Solution:

3) Determine the co-ordinates of the point on the curve:

$y = 2x^2 - 3x - 5$, where the gradient is 1.

Solution:

4) If $y = \frac{6\,cos\,5x}{x^5}$, determine $\frac{dy}{dx}$

Solution:

5) Determine the rate of change of voltage, given $v = 5t\ sin\ 2t$ *volts*, when $t = 0.2$

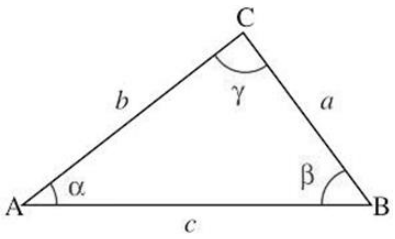
(Round off answer to 3 significant figures)

Solution:

END OF QUESTIONS

Formula Sheet

Law of Sines and Cosines



$$\frac{\sin\alpha}{a}=\frac{\sin\beta}{b}=\frac{\sin\gamma}{c}$$

$$a^2=b^2+c^2-2bc\cos\alpha$$

$$b^2=a^2+c^2-2ac\cos\beta$$

$$c^2=a^2+b^2-2ab\cos\gamma$$

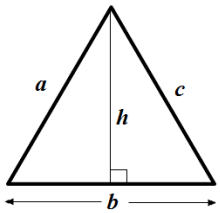
$$cos\propto=\frac{b^2+c^2-a^2}{2bc}$$

$$cos\beta=\frac{a^2+c^2-b^2}{2ac}$$

$$cos\gamma=\frac{a^2+b^2-c^2}{2ab}$$

Perimeter, Area and Volume

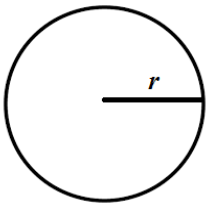
Triangle:



$$P=a+b+c$$

$$A=\frac{1}{2}bh$$

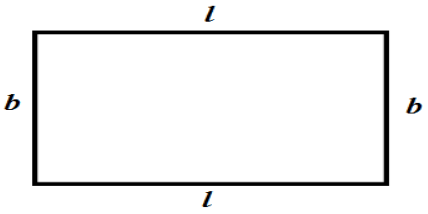
Circle:



Circumference $(C)=2\pi r=\pi d$

$$A=\pi r^2$$

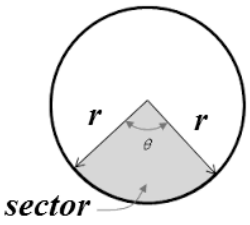
Rectangle:



$$P=2l+2b$$

$$A=lb$$

Sector:



Length of the arc:

$L=\theta r$ if θ is in radians

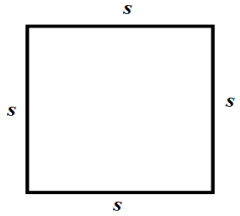
$L=\theta\left(\frac{\pi}{180}\right)r$ if θ is in degrees

Area: $A=\frac{1}{2}Lr$

$A=\frac{1}{2}\theta\ r^2$ if θ isinradians

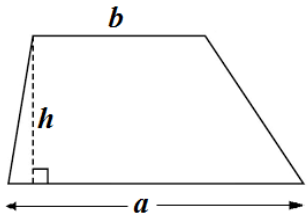
$A=\theta\left(\frac{\pi}{360}\right)r^2$ if θ isindegrees

Square:



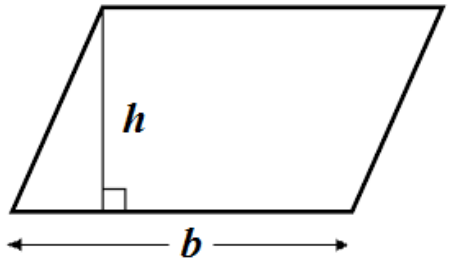
$P=4s$ $A=s^2$

Trapezium:



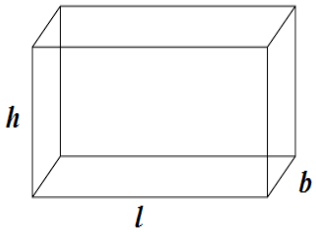
$$A = \frac{1}{2}(a+b)h$$

Parallelogram:



$$A = bh$$

Cuboid:

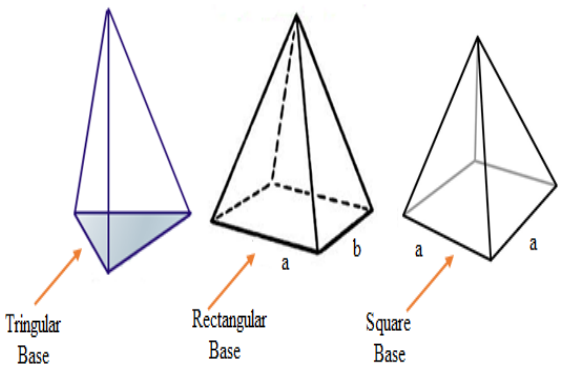


$$V = lbh$$

$$LSA = 2h(l+b)$$

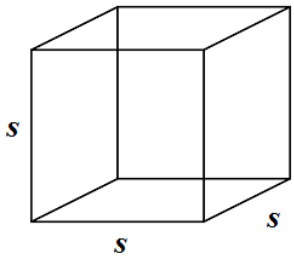
$$TSA = 2(lb+bh+hl)$$

Pyramid:



$$V = \frac{1}{3}\,Area\,of\,the\,base \times Height = \frac{1}{3}Ah$$

Cube:

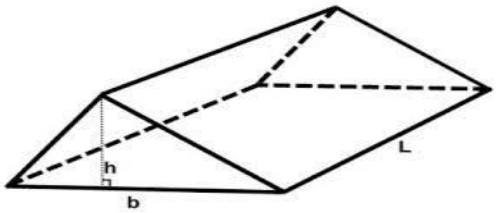


$$V = S^3$$

$$LSA = 4s^2$$

$$TSA = 6s^2$$

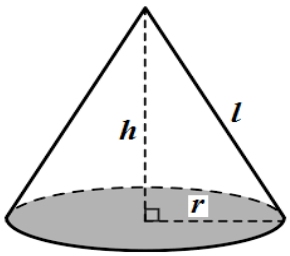
Prism:



$$V = Area\,of\ cross\ section \times Length$$

$$= \left(\frac{1}{2}bh\right)L$$

Cone:



$$V = \frac{1}{3}\pi r^2 h$$

$$CSA = \pi r l$$

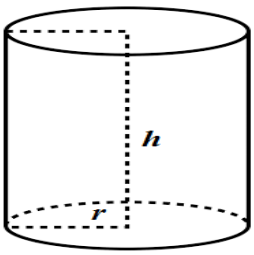
$$TSA = \pi r^2 + \pi r l$$

Cylinder:

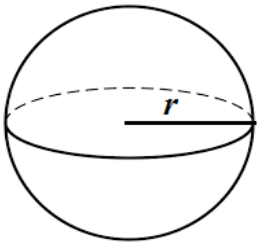
$$V = \pi r^2 h$$

$$CSA = 2\pi r h$$

$$TSA = 2\pi r(r+h)$$



Sphere:



$V = \frac{4}{3}\pi r^3$

$CSA = TSA = 4\pi r^2$

Statistics

$Relative\ frequency = \frac{f\ of\ the\ class}{\sum f}$

$\theta = relative\ frequency \times 360^\circ$

standard deviation $= \sqrt{\text{Variance}}$

For ungrouped data

Mean = $\bar{x} = \frac{\sum x}{n}$

Sample Variance = $s^2 = \frac{\sum(x_i - \bar{x})^2}{n-1}$

$$\textbf{or} \quad s^2 = \frac{n\sum x_i^2 - (\sum x_i)^2}{n(n-1)}$$

For grouped data

Mean = $\bar{x} = \frac{\sum f_i x_i}{\sum f_i}$

Median = $L_m + \left[\frac{\frac{N}{2} - cf_m}{f_m}\right] i$

Where, L_m = lower class boundary of the median class

N = the number of cases (items) in the set.

cf_m = the cumulative frequency before the median class.

f_m = frequency of the median class

i = class width or class size

Mode = $L_{mo} + \left[\frac{\Delta_1}{\Delta_1 + \Delta_2}\right] i$

Where, L_{mo} = lower class boundary of the modal class

Δ_1 = the difference between the frequency of the modal class and the frequency of the class **before** the modal class.

Δ_2 = the difference between the frequency of the modal class and the frequency of the class **after** the modal class.

i = class width or class size

Sample Variance = $s^2 = \frac{\sum f_i(x_i - \bar{x})^2}{n-1}$

$$\textbf{or} \quad s^2 = \frac{n\sum f_i x_i^2 - (\sum f_i x_i)^2}{n(n-1)}$$

Probability

1) If an experiment can result in any one of N different equally likely outcomes, and if exactly n of these outcomes corresponds to event A, then the probability of event A is given by $P(A) = \frac{n}{N}$

2) The number of permutations of n distinct objects is n!

3) The number of permutations of n distinct objects taken r at a time is ${}_nP_r$

4) The number of permutations of n distinct objects arranged in a circle is (n – 1)!

5) The number of combinations of n distinct objects taken r at a time is: ${}_nC_r$

6) If A and B are any two events, then $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

7) If A and B are two mutually exclusive events, then $P(A \cup B) = P(A) + P(B)$

8) If A and A' are complementary events, then $P(A) + P(A') = 1$

9) If in an experiment, the events A and B can both occur, then
$P(A \cap B) = P(A).P(B/A)$

10) If two events A and B are independent, then $P(A \cap B) = P(A).P(B)$.

Properties of exponential function

1) $a^x a^y = a^{x+y}$

2) $(a^x)^y = a^{xy}$

3) $(ab)^x = a^x b^x$

4) $\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$

5) $\frac{a^x}{a^y} = a^{x-y}$

6) $a^x = a^y$ if and only if $x = y$

7) $a^x = b^x$ if and only if $a = b$

Definition of logarithmic function

$y = \log_a x \Leftrightarrow x = a^y$

Properties of Logarithms

1) $\log_a(xy) = \log_a x + \log_a y$

2) $\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$

3) $\log_a x^b = b\log_a x$

Quadratic Equation
Solution of $ax^2 + bx + c = 0$ is given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Rules Of Differentiation

1. $\frac{d(c)}{dx} = 0$ where c is any constant.

2. $\frac{d}{dx}[a.f(x))] = a.\frac{d\,f(x)}{dx}$

3. $\frac{d(x^n)}{dx} = nx^{n-1}$

4. $\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$

5. $\frac{d}{dx}[f(x).g(x)] = f(x).g'(x) + g(x).f'(x)$

or

$\frac{d}{dx}[u.v)] = u.\frac{dv}{dx} + v.\frac{du}{dx}$ where u and v are two different functions of x.

6. if $y = \frac{f(x)}{g(x)}$,

then $\frac{dy}{dx} = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$

General Power form:

$\frac{d(u^n)}{dx} = nu^{n-1}\frac{d\,(u)}{dx}$ where $u = f(x)$.

Derivatives of Trigonometric Functions

1. $\frac{d}{dx}(sin\,u) = cos\,u\,.\frac{d\,(u)}{dx}$ where $u = f(x)$.
2. $\frac{d}{dx}(cos\,u) = -sin\,u\,.\frac{d\,(u)}{dx}$
3. $\frac{d}{dx}(tan\,u) = sec^2\,u\,.\frac{d\,(u)}{dx}$

Derivatives of Exponential Functions

Let ***a*** be any real number but not zero and

$u = f(x)$

1. $\frac{d}{dx}(a^u) = a^u\,lna.\frac{d\,(u)}{dx}$
2. $\frac{d}{dx}(e^u) = e^u\,.\frac{d\,(u)}{dx}$

Derivatives of Logarithmic Functions

Let ***a*** be any real number but not zero and

$u = f(x)$

1. $\frac{d}{dx}(\log_a u) = \frac{1}{u\,lna}\,\frac{d\,(u)}{dx}$
2. $\frac{d}{dx}(ln\,u) = \frac{1}{u}\,\frac{d\,(u)}{dx}$

ANSWERS

SECTION A

Q #	Answer
1	a
2	c
3	b
4	c
5	a

Q #	Answer
6	a
7	c
8	b
9	a
10	a

Q #	Answer
11	c
12	b
13	a
14	b
15	c

Q #	Answer
16	b
17	a
18	c
19	c
20	a

SECTION B

1) $40.4°$
2) 6
3) 26.5
4) 0.6
5) 28
6) 7400
7) 0.3
8) $\frac{1}{2}$
9) 10
10) $5(3x^2 - 2x)^4(6x - 2)$

SECTION C

1) $\frac{11}{3}$
2) 1.1
3) $(1, -6)$
4) $\frac{-30x^5 sin5x - 30x^4 cos\,5x}{x^{10}}$
5) 2.03

DRAFT / ROUGH WORK