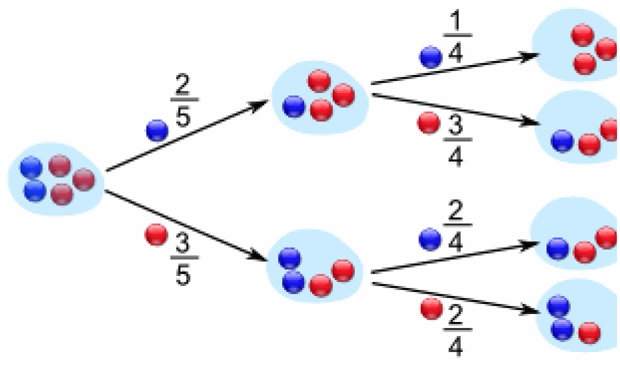
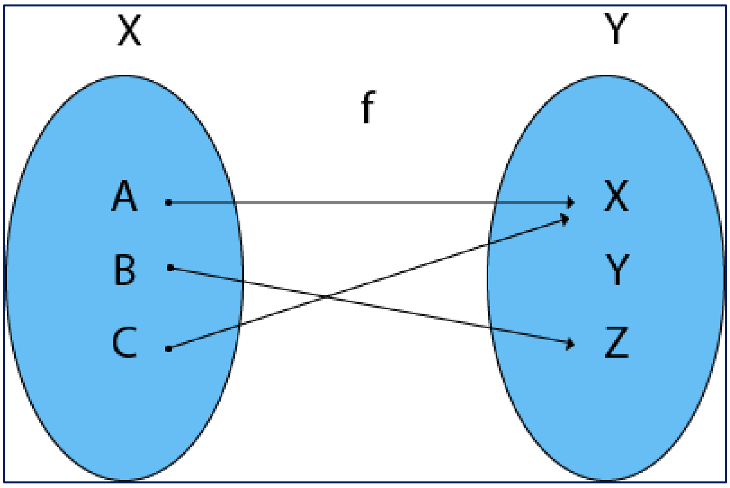
Military Technological College





GFP- Pure Mathematics

MODULE CODE: MTCG1018

WORKBOOK- 2

TERM-2 **AY: 2023-2024** **GFPMATH2**

	Learning Outcomes – On successful completion of this module, students should be able to:
1.	Demonstrate understanding of the definition of a function and its graph.
2.	Define and manipulate exponential and logarithmic functions and solve problems arising from real life applications.
3.	Understand the inverse relationship between exponents and logarithms functions and use this relationship to solve related problems.
4.	Understand basic concepts of descriptive statistics, mean, median, mode and summarize data into tables and simple graphs (bar charts, histogram, and pie chart).
5.	Understand basic probability concepts and compute the probability of simple events using tree diagrams and formulas for permutations and combinations.
6.	Define and evaluate limit of a function as well as test continuity of a function.
7.	Determine the surface areas, the volumes and capacities of common shapes and 3-dimesions figures (square, rectangle, parallelogram, trapezium, cuboid, cone, pyramid and prisms).
8.	Find the derivatives of standard and composite functions using standard rules of differentiation.
9.	Use the law of sines and cosines to solve a triangle and real-life problems.

MILITARY TECHNOLOGICAL COLLEGE

Delivery Plan - Year 2023-24
[Term 2]

Title / Module Code / Programme	Pure Mathematics /MTCG1018/Foundation Programme Department (FPD)	Module Coordinator	Mr. Knowledge Simango
Lecturers	TBA	Resources & Reference books	Moodle & Workbook
Duration & Contact Hours	Term 2: 4 hrs x 11 weeks = 44 hours		

Week No.	TOPICS	Hours	Learning Outcome No.
1	**Introduction** **1. Law of sines and cosines to solve a triangle** 1.1 Law of sines 1.2 Law of cosines **2. Perimeter, Area and Volume** 2.1 Perimeter and area	4	7, 9
2	2.2 Volume and surface area **3. Statistics** 3.1 Basic concepts of descriptive statistics 3.2 Types of Data **Revision for Continuous Assessment-1**	4	4, 7
	Continuous Assessment-1 (Chapter 1 and 2)		7 and 9
3	3.3 Summarizing and presenting data. 3.4 Measures of Central Tendency 3.5 Measures of Dispersion	4	4
4	**4. Probability** 4.1 Basic Concepts 4.2 Probability 4.3 Rules of Probability	4	5
5	**5. Functions and graphs** 5.1 Domain, range and function 5.2 Types of functions 5.3 Inverse function	4	1

6	5.4 Operations of functions 5.5 Composite function **6. Exponential functions** 6.1 Exponential equations	**4**	**1**
7	6.2 Exponential function and graphs 6.3 Application in real life **Revision for Continuous Assessment-2**	**4**	**2**
	Continuous Assessment-2 (Chapter 3, 4 and 5)		**1, 4 and 5**
8	**7. logarithmic functions** 7.1 Logarithm Definition and Properties 7.2 Logarithmic function and graph 7.3 Exponential and logarithmic equations **8. Limits** 8.1 Basic Concepts of Limit	**4**	**2, 3, 6**
9	8.2 Methods of finding limits 8.3 Limits at Infinity 8.4 Continuity of a Function **9. Differentiation** 9.1 The Gradient of a Curve	**4**	**6, 8**
10	9.2 Differentiation from the First Principles 9.3 Methods of Differentiation	**4**	**8**
11	9.4 Applications of Derivatives	**4**	**8**
	Revision for Final Exam,		**1, 2, 3, 8 & 9**
12/13	**FINAL EXAM (Unit-6 to Unit-9)**		**1, 2, 3, 8 & 9**
	Total hours	**44**	

Indicative Reading	
Title/Edition/Author	**ISBN**
College Algebra with Trigonometry-7th Edition by K Raymond A., Ziegler Michael R., Byleen	ISBN-13: 978-0072368697 ISBN-10: 0072368691
College Algebra and Trigonometry-5th Edition by Margaret L. Lial, John Hornsby, David I. Schneider and Callie Daniels	ISBN-13: 978-0321671783 ISBN-10: 0321671783
Bird's Basic Engineering Mathematics- 8th Edition by John Bird	ISBN-13: 978-0367643706 ISBN-10: 0367643707
Engineering Mathematics- 8th Edition by K.A. Stroud and Dexter Booth	ISBN-13: 978-1352010275 ISBN-10: 1352010275

Mr. Knowledge Simango
Module Coordinator

Dr. T Raja Rani
DHOD FPD(CMP)

MQM Salim Al Shibli
Head FPD

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<u>Assessment Plan</u> (Passing Mark: 50 %)

Assessment	Weightage
Continuous Assessment-1	20%
Continuous Assessment-2	30%
Final Exam	50%
Total	100%

Note: Only Non-Programmable calculators, prescribed in MTC exam rules, are allowed.

<u>Attendance Policy:</u>

Warning	Absence
First	10%
Second	15%
Third	20%

(UNIT-3) STATISTICS

3.1 BASIC CONCEPTS OF DESCRIPTIVE STATISTICS

1) Fundamental Concepts

1.1) Data: A collection of numbers or facts that are used as a basis for making conclusions.

1.2) Statistics: It is the science of collecting, summarizing, and analyzing numerical data. Statistics makes it possible to predict the likelihood of events.

Depending upon the use of the data the study of statistics is divided into two main areas.

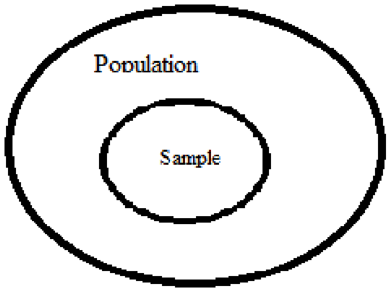
Descriptive Statistics and Inferential Statistics

a) Descriptive statistics consists of the collection, organization, summarization, and presentation of the data so as to yield meaningful information. In descriptive statistics, the statistician tries to describe a situation.

b) Inferential statistics consists of generalizing from samples to populations, performing estimations and hypothesis tests, determining relationships among variables and making predictions. Here the statistician tries to make inferences from samples to populations.

1.3) Population: A population consists of all subjects (human or otherwise) that are being studied. Examples:
1) The whole student body at MTC.
2) All Jet engines produced this year.

1.4) Sample: A sample is a group of subjects selected from a population. Examples:
1) A group of 50 student representatives selected from MTC for survey.
2) A few Jet engines selected from all produced in a year, for a testing.

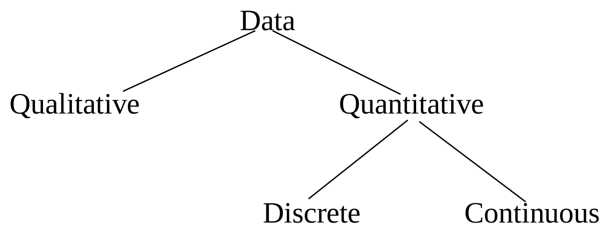


Note: Samples are used due to expense, time, size of population and other concerns.

1.5) Sample Size: It is the number of elements or observations chosen randomly from a population.

2) Types of Data

The data can be classified as follows:



2.1) Qualitative or categorical data are descriptive and represented by words or symbols that characterize a set of values.

Example: colors, shapes, nationalities, brand names and so on.

Brand of T-Shirts	**Number of Pieces Sold in 1 week**
Nike	15
Bossini	10
Gap	20
Mark and Spencer	12

2.2) Quantitative data are numerical and can be ranked or ordered.

a) Quantitative Discrete data are measurements or values obtained by counting and are recorded using whole numbers only. The set of discrete data is finite and countable.

Example: The number of students enrolled in Math, Physics, and Computing subjects:

Subjects	**Number of Students Enrolled**
Basic Math	250
Physics	300
Computing	220

b) C**ontinuous dat**a are values within a bounded or boundless interval. Between any two continuous data values, there may be an infinite number of others. Continuous data is obtained by measuring.

Example: height, weight, distance, time, temperature, etc.

Weight (kilograms)	Number of Pieces
7 - 9	2
10 - 12	8
13 - 15	14
16 - 18	19
19 - 21	7
	n = 50

Class Activity

A) **Circle the correct answer for each question**.

1) The group of all subjects under a study is called _____.

a) Sample
b) Population
c) Statistics

2) A group of subjects selected from the group of all subjects under study is called_______ .

a) Sample
b) Population
c) Statistics

3) Which kind of data that are non-numeric?

a) Qualitative
b) Quantitative Continuous
c) Quantitative discrete

4) Data obtained by counting and recorded using whole numbers are called________ data.

a) Qualitative
b) Quantitative discrete
c) Quantitative Continuous

B) Classify each variable as quantitative-discrete **or** quantitative-continuous **or** qualitative data.

1) Colours of automobiles in the faculty parking lot.

Ans)

2) Number of desks in classrooms.

Ans)

3) Weights of fish caught in a lake.

Ans)

4) Number of pages in statistics textbooks.

Ans)

5) Life time of flash light batteries.

Ans)

3.2 SUMMARIZING AND PRESENTING DATA

1) Tabular Presentation of Data

Data can be organized in tabular form, commonly called as frequency distribution table.

The reasons for constructing a frequency distribution are

1) To organize the data in a meaningful, intelligent way.
2) To enable the reader to determine the nature or shape of the distribution.
3) To facilitate computational procedures for measures of averages and spread.
4) To enable the researcher to draw charts and graphs for the presentation of the data.
5) To show the reader to make comparisons among different data sets.

1.1 Frequency Distribution for Qualitative Data

Example 1: .Let's suppose you make a survey concerning favorite color, and the data you collect looks something like the table below.

blue	red	blue	orange	blue	yellow	green	red	pink
blue	green	blue	purple	blue	blue	green	yellow	pink
blue	red	pink	green	blue	yellow	green	blue	

Organize and present the data in a frequency distribution table.
Solution:

Favorite color	frequency
blue	10
red	3
orange	1
yellow	3
green	5
pink	3
purple	1

Class Activity-1

1) Construct a frequency distribution for these network data.

CBS	Fox	ABC	Fox
CBS	CBS	ABC	Fox
CBS	ABC	CBS	CBS
NBC	CBS	NBC	NBC
CBS	CBS	NBC	NBC

Solution:

1.2 Constructing Cumulative Frequency Distribution for Quantitative Data (Discrete or Continuous)

1) Frequency Distribution: For constructing a frequency distribution follow these steps:

Step 1) Identify the highest and the lowest value in the data.

Step 2) Create a column (or row) with the title of the variable. Enter the lowest score at the top and include all the values from lowest to the highest value.

Step 3) Create a frequency column (or row), with the frequency of each value.

Step 4) At the bottom (or end of row) of the frequency column record the total frequency for the distribution proceeded by N or ∑ f

Step 5) Enter the name of the frequency distribution at the top of the table.

Example 1: Summarize the following data of high temperatures for 30 days, by creating a frequency distribution.

50	45	49	50	43	49
50	49	45	49	47	47
44	51	51	44	47	46
50	44	51	49	43	43
49	45	46	45	51	46

Solution: Following is frequency distribution table showing high temperatures for 30 days:

Temperature	**Frequency(f)**
43	3
44	3
45	4
46	3
47	3
48	0
49	6
50	4
51	4
	∑f =30

2) Cumulative frequency distribution: The cumulative frequency for each value is the frequency up to and including the frequency of that score.

Step 1) Create a frequency distribution.

Step 2) Add a column entitled cumulative frequency. The highest cumulative frequency should equal to N or ∑ f, that is, the total of frequency column.

Example 1: Summarize the data from the previous table of high temperatures for 30 days, by creating a cumulative frequency distribution.

Solution:

Temperature	**Frequency (f)**	**Cumulative frequency(cf)**
43	3	3
44	3	6
45	4	10
46	3	13
47	3	16
48	0	16
49	6	22
50	4	26
51	4	30

Class Activity-2

1) The following figures represent the number of children born to 50 women in a certain locality up to the age of 40 years.

1	5	1	0	2	5
9	2	6	3	5	7
8	4	6	8	9	10
9	3	5	7	9	9
4	5	4	5	5	7
3	4	2	3	4	6
3	4	2	5	6	4
0	5	6	8	5	4
7	6				

If the researcher wants 0 to 10 to be separate categories, then there are 11 classes and the class size is 1. Construct a cumulative frequency distribution table.

Solution: The frequency distribution of such data is as follows:

Number of Children	Tally	Number of Women (*f*)	cf
0			
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			
Total		50	

3) Grouped cumulative frequency distribution

Procedure for constructing a grouped cumulative frequency distribution:

Step 1) Determine the classes:

a) Find the highest and the lowest value
b) Find the range= Highest value – lowest value
c) Select the number of classes desired. For a small set of numbers use a minimum of 5 classes. For a large number of cases, calculate K (number of classes) using this rule:
$\boldsymbol{K} = \sqrt{\boldsymbol{N}}$ where $\boldsymbol{N}$ is the number of cases or items in the set. **Always round up *K* to the next whole number.** (rounding up is different from rounding off for example 14.167 = 15 and 13.25 = 14 and 4.9 = 5).
In general, we have **5 to 20** classes.
d) Find the width or class interval size by dividing the range by the number of classes and rounding up
e) Select a starting point (usually the lowest value or any convenient number less than the lowest value). Add the width to get the lower limits
f) Find the upper limits. (Subtract one unit from the lower limit of the second class to get the upper limit of the first class. Then add the width to each upper limit to get all the upper limits)

Step 2) Tally the data (it is an optional step to help in finding frequency)

Step 3) Find the numerical frequencies from the tallies.

Step 4) Find the cumulative frequencies.

Note: To construct only Grouped frequency distribution we can avoid step-4

Example 1) These data represent the record high temperature for each of the 50 states. Construct a grouped cumulative frequency distribution for the data using 7 classes.

112	100	127	120	134
110	118	117	116	118
107	112	114	115	118
116	108	110	121	113
120	113	120	117	105
118	105	110	109	112
122	114	114	105	109
117	118	122	106	110
120	119	111	104	111
110	118	112	114	114

Solution:

The highest value=H=134

and lowest value=L=100.

The range= H-L =134-100 =34

Since the number of classes is given as 7, the class width $= \frac{34}{7} = 4.9 \approx 5$

If we select 100 to be the starting point then our class limits are: 100,105,110,115,120,125,130 and 135

Subtracting 1 from the lower class limit of the 2nd class to get the upper limit of 1st class: 105-1=104

we have the classes as 100-104, 105-109, …..

The completed distribution table is shown below:

Class interval	**Frequency**	**Cumulative frequency**
100-104	2	2
105-109	8	10
110-114	18	28

115-119	13	41
120-124	7	48
125-129	1	49
130-134	1	50

Class Activity-3

1) At a recent chess tournament, all 10 of the participants had to fill out a form that gave their names, address, and age. The ages of the participants were recorded as follows: **36, 48, 54, 92, 57, 63, 66, 76, 66,** and **80.** Construct a grouped cumulative frequency distribution of **10 classes**.

Solution:
Here is the frequency distribution for the ages of the participants:

Class Intervals	Tally	Frequency	**Cumulative frequency**
		$\sum f =$	

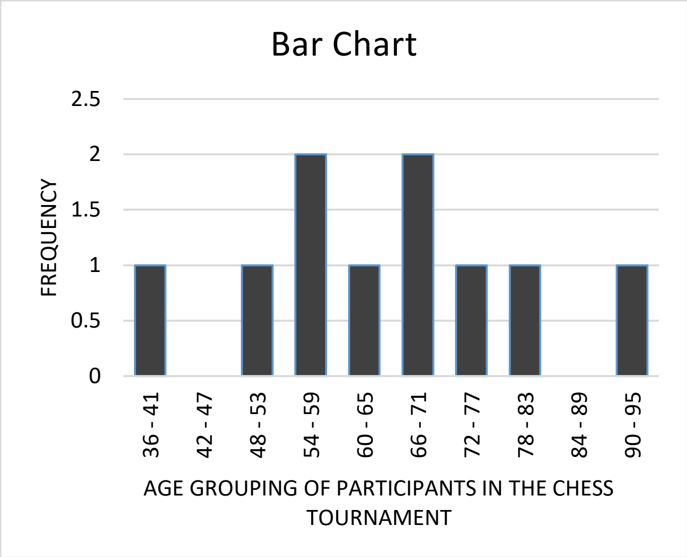
2) Graphical Presentation of Data

After the data have been organized into a frequency distribution, they can be presented in graphical form. The purpose of graphs in statistics is to convey the data to the viewers in pictorial form. It is easier for most people to comprehend the meaning of data presented graphically than data presented numerically in tables or frequency distributions. This is especially true if the users have little or no statistical knowledge.
Statistical graphs can be used to describe the data set or to analyze it. Graphs are also useful in getting the audience's attention in a publication or a speaking presentation. They can be used to discuss an issue, reinforce a critical point or summarize a data set. They can also be used to discover a trend or pattern in a situation over a period of time.
The most commonly used forms of data presentation are:
a) Bar (Column) Chart,
b) Histogram, and
c) Pie Chart.

a) Bar (Column) Chart

For the **bar chart,** Categorical data/class intervals are used on the x – axis and the frequencies are used on the y –axis.
There are gaps between the columns or bars.
Example 1: Construct a **bar chart** for the given frequency distribution:

Age Group	Frequency f
36 - 41	1
42 - 47	0
48 - 53	1
54 - 59	2
60 - 65	1
66 - 71	2
72 - 77	1
78 - 83	1
84 - 89	0
90 - 95	1



<u>Class Activity-4</u>

1) Construct a bar chart for the data below:

Weight (kilograms)	Number of Pieces
7 - 9	2
10 - 12	8
13 - 15	14
16 - 18	19
19 - 21	7
	n = 50

Solution:

<u>b) Histogram</u>

A histogram is a column chart. The bases of the columns represent the **class boundaries** and the heights of the columns indicate the **frequencies**. There are no gaps in between the columns.

The class boundaries are put on the *x*–axis and the frequencies are on the *y*–axis.

Method to find class boundaries: Find the difference of the first upper limit and the second lower limit and divide by 2. Then find the class boundaries by subtracting that value from each lower limit and adding that value to each upper–class limit. For example if first class is 100 – 104 and the second class is 105 –109 then 105–104 = required difference=1 and required value ½ = .5 so class boundaries are 99.5 – 104.5 , 104.5 – 109.5 etc)

or if the class limits are in whole number subtract and add 0.5 to the lower and upper limits respectively. If the limits are in first decimal subtract and add 0.05 to the lower and upper limits respectively. This can be generalized.

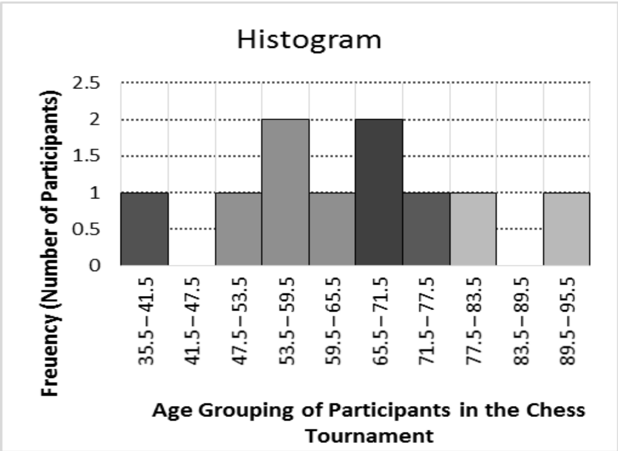
For example if we subtract 0.5 from each lower class limit and add 0.5 to each upper class limit, we have the following class boundaries : 99.5–104.5, 104.5–109.5,…

Example 1: Construct a **histogram** for the given frequency distribution:

Class Intervals	Frequency f
36 - 41	1
42 - 47	0
48 - 53	1
54 - 59	2
60 - 65	1
66 - 71	2
72 - 77	1
78 - 83	1
84 - 89	0
90 - 95	1
	$\sum f = 10$

Solution:

Class Intervals	Class Boundaries	Frequency f
36 - 41	35.5 – 41.5	1
42 - 47	41.5 – 47.5	0
48 - 53	47.5 – 53.5	1
54 - 59	53.5 – 59.5	2
60 - 65	59.5 – 65.5	1
66 - 71	65.5 – 71.5	2
72 - 77	71.5 – 77.5	1
78 - 83	77.5 – 83.5	1
84 - 89	83.5 – 89.5	0
90 - 95	89.5 – 95.5	1
		$\sum f = 10$



Class Activity-5

Section –A Circle the correct answer in the following questions:

1) The class boundaries of the interval 13 – 19 is ………
(a) 12 - 20
(b) 13.5 – 19.5
(c) 12.5 – 19.5

2) The class boundaries of the interval 6.3 – 9.7 is ………
(a) 6.35 - 9.75
(b) 6.2 - 9.6
(c) 6.25 – 9.75

3) If the class boundaries are 6.45 - 9.55, then the class interval is ………
(a) 6.5 – 9.5
(b) 7.0 - 9.0
(c) 6.0 – 9.0

Section-B

1) Find class boundaries of following class intervals.

a) 11-15

Answer:

b) 11.8-14.7

Answer:

c) 3.13-3.93

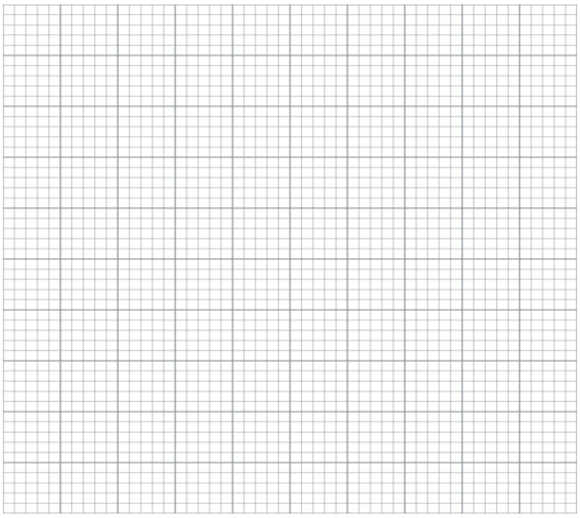
Answer:

2) For 108 randomly selected college applicants, the following frequency distribution for entrance exam scores were obtained:

Class Interval	Frequency
90-98	6
99-107	22
108-116	43
117-125	28
126-134	9

Construct a histogram for the data.

Solution:



Pie charts display categorical or grouped data. These charts are commonly used within the industry to communicate simple ideas. They are used to show the proportions of a whole. They are best used when there are only a handful of categories to display.

A pie chart consists of a circle divided into segments, one segment for each category. The size of each segment is determined by the frequency of the category and measured by the angle of the segment. As the total number of degrees in a circle is 360, the angle given to a segment is 360° times the fraction of the data in the category, that is the central angle θ, in degrees, for each sector is determined by the following rule:

$$\theta = \frac{f \text{ of the class}}{\sum f} \times 360°$$

Where $\frac{f \text{ of the class}}{\sum f} = relative\ frequency(rf)$

The relative frequency can also be written in percent form.

Example 1: In a certain intake at MTC, there are 800 students distributed as follows; 200 Aeronautical Engineering students, 250 Civil Engineering students, 220 Systems Engineering and 130 Marine Engineering students. Show the data using a pie chart.

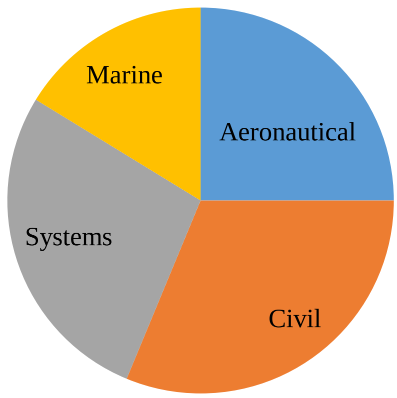
Solution:

We start by calculating the angles that will proportionally represent the students in the different departments.

Total Number(n) =800

Branch	**No. of Students**	$\theta = rf \times 360°$
Aeronautical	200	$\frac{200}{800} \times 360° = 90°$
Civil	250	$\frac{250}{800} \times 360° = 112.5°$
Systems	220	$\frac{220}{800} \times 360° = 99°$
Marine	130	$\frac{130}{800} \times 360° = 58.5°$
	800	

Now we can draw a circle using a compass and use a protractor to measure the angles so that the sectors representing the data can be shown.



Different colors are used to distinguish the sectors of our pie chart.

Note: Pie charts are good for displaying data for around 6 categories or fewer. When there are more categories it is difficult for the eye to distinguish between the relative sizes of the different sectors and so the chart becomes difficult to interpret

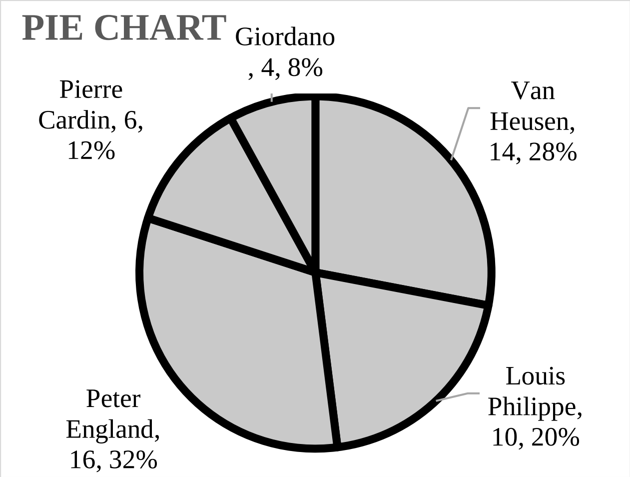
Example 2: A survey was done involving 50 male shoppers in Grand Mall and they were asked to state the brand of shirt they like most. The data has been summarized as follows:

Brand of Shirt	**No. of Male Shoppers**
Van Heusen	14
Louis Philippe	10
Peter England	16
Pierre Cardin	6
Giordano	4

Construct a pie chart for this data.

Solution:

Brand of Shirt	**No. of Male Shoppers**	rf	θ
Van Heusen	14	0.28	100.8°
Louis Philippe	10	0.2	72°
Peter England	16	0.32	115.2°
Pierre Cardin	6	0.12	43.2°
Giordano	4	0.08	28.8°



Class Activity-6

Circle the answer in each question.

1) Given the frequency distribution of a sample of 50 pieces of luggage inspected randomly in an airport.

Weight (kilograms)	Number of Pieces
7 - 9	2
10 - 12	8
13 - 15	14
16 - 18	19
19 - 21	7
	n = 50

i) The class size of the distribution is________.

(a) 2
(b) 3
(c) 5

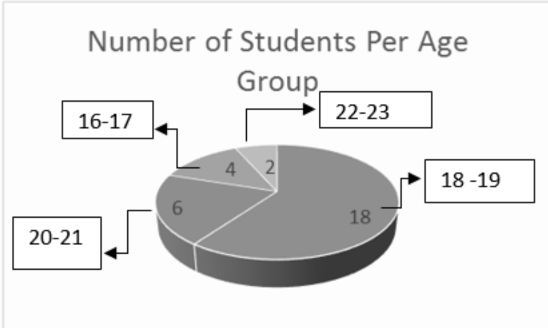
ii) The number of pieces of luggage weighing less than 12.5 kilograms is_______.

(a) 10
(b) 8
(c) 14

iii) The relative frequency of a luggage whose weight is 13 to 15 kg is __________.

(a) 24
(b) 14
(c) 0.28

2) Given the pie graph below:



i) How many students are there in the sample?

(a) 20
(b) 40
(c) 30

ii) Which age group has the highest percentage?

(a) 20-21
(b) 18 - 19
(c) 22-23

iii) What % is the age group 20 – 21?

(a) 20%
(b) 10%
(c) 30%

iv) What is the measure of the central angle occupied by the sector of age group 18 – 19?

(a) 120^0
(b) 180^0
(c) 216^0

3.3 MEASURES OF CENTRAL TENDENCY

A ***measure of central tendency*** gives a single value that acts as a representative or average of all the values in a set. There are three measures of central tendency, namely: **mean, median,** and **mode.**

1) Mean ($\overline{x}$)

The **arithmetic mean value** is found by adding together the values of the members of a set and dividing by the number of members in the set. The ***arithmetic mean*** is synonymous to the word "**average**".

$$\text{Mean}= \overline{x} = \frac{\sum \boldsymbol{x}}{\boldsymbol{n}}$$

A) Mean of Ungrouped Data

In general, for any given set of numbers $\{x_1; x_2; x_3 \ldots, x_n\}$ the mean is given by:

$$\overline{x} = \frac{x_1+x_2+x_3+\cdots+x_n}{n} = \frac{\sum \boldsymbol{x}}{\boldsymbol{n}}$$

Example 1: Find the mean of the following numbers; 1, 3, 5, 7 is $\frac{1+3+5+7}{4} = 4$

Example 2: The number of students attending Math class in 5 days is as follows: 15, 18, 17, 15, and 12. Find the mean number of students attending Math class per day.

Solution:

$$\overline{x} = \frac{15+18+17+15+12}{5} = 15.4$$

$$\overline{x} = 15\ students$$

B) Mean of Grouped Data

The mean of a frequency distribution or grouped data can be calculated as:

$$\overline{x} = \frac{f_1x_1 + f_2x_2 + \cdots + f_nx_n}{f_1 + f_2 + \cdots f_n} = \frac{\sum(\boldsymbol{f_i x_i})}{\sum \boldsymbol{f_i}}$$

where $\boldsymbol{f_i}$ is the frequency and $\boldsymbol{x_i}$ is the mid-point of the class intervals.

Example 1: The table below shows the distribution of marks obtained by a group of 300 students over a 100-mark examination.

Marks (x)	Number of Students (f)
11 - 20	40
21 - 30	45
31 - 40	60
41 - 50	55
51 - 60	40
61- 70	31
71 - 80	14
81 - 90	10
91 - 100	5

Find the average mark of the students to the nearest whole number.

Solution:

Marks (x)	Midpoint x_i	Number of Students (f)	$f_i x_i$
11 - 20	(11+20)/2 =15.5	40	40 x 15.5 =620
21 - 30	25.5	45	1147.5
31 - 40	35.5	60	2130
41 - 50	45.5	55	2502.5
51 - 60	55.5	40	2220
61- 70	65.5	31	2030.5
71 - 80	75.5	14	1057
81 - 90	85.5	10	855
91 - 100	95.5	5	477.5
		$\sum f = 300$	$\sum fx =$ 13040

$$\overline{x} = \frac{\sum(fx)}{\sum f} = \frac{13040}{300} = 43.47 = 43$$

Class Activity 1

1) The following are the numbers of twists that were required to break 12 forged alloy bars: 33, 24, 39, 48, 26, 35, 38, 54, 23, 34, 29 and 37.
Find the mean.
Solution:

2) Find the mean of the distribution:

Class Intervals	Frequency f		
36 - 41	1		
42 - 47	0		
48 - 53	1		
54 - 59	2		
60 - 65	1		
66 - 71	2		
72 - 77	1		
78 - 83	1		
84 - 89	0		
90 - 95	1		
	$\sum f = 10$		

Solution:

2) Median

The **median** is the middle value of an ordered data set. The median value is obtained by:

Step 1) ranking/sorting the data in ascending or descending order of magnitude, and

Step 2) selecting the value of the middle member for sets containing an odd number of members, or finding the value of the mean of the two middle members for sets containing an even number of members

The **median** value often gives a better indication of the general size of a set containing extreme values. The set: {7, 5, 74, 10} has a mean value of 24, which is not really representative of any of the values of the members of the set. The **median is the most stable value** among the measures of central tendency.

A) Median of Ungrouped Data

Example 1: Find the median of each of the following distributions:

i) 44, 49, 52, 62, 53, 48, 54, 46, 51

ii) 40 , 45 , 12 , 6 , 9 , 16 , 11 , 7 , 35 , 7 , 31 ,3

Solution:

i) The re-arranged or ordered distribution is:

44, 46 , 48, 49, 51, 52, 53, 54, 62

Note that $n = 9$ and $\frac{n}{2} = 4.5$, then the 5^{th} value is the median.

The median is 51.

ii) The re-arranged or ordered list is : 3, 6, 7, 7, 9, 11, 12, 16, 31, 35, 40, 45

Since the list contains an even number of values the two middle elements are 11 and 12, so the median $= \frac{11+12}{2} = 11.5$

B) Median for Grouped Data

Procedure:

Step 1) Construct cumulative frequency distribution.

Step 2) Determine the median class. It is the class interval with cumulative frequency counted at which the $\frac{N}{2}^{th}$ position is included.

Step 3. Calculate the median by this rule:

$$\text{Median} = L_m + \left[\frac{\frac{N}{2} - cf_m}{f_m}\right] i$$

where, L_m = lower class boundary of the median class

N = the number of cases (items) in the set.

cf_m = the cumulative frequency before the median class.

f_m = frequency of the median class

i = class size

Example 1: The table below shows the distribution of marks obtained by a group of 300 students in a 100-mark examination.

Marks (*x*)	Number of Students (*f*)
11 - 20	40
21 - 30	45
31 - 40	60
41 - 50	55
51 - 60	40
61- 70	31
71 - 80	14
81 - 90	10
91 - 100	5
	N=300

Find the median.

Solution:

Marks (*x*)	Class Boundaries	Number of Students (*f*)	Cum Freq (*cf*)
11 - 20	10.5 – 20.5	40	40
21 - 30	20.5 – 30.5	45	85
31 - 40	30.5 – 40.5	60	145
41 - 50	40.5 – 50.5	55	200
51 - 60	50.5 – 60.5	40	240
61- 70	60.5 – 70.5	31	271
71 - 80	70.5 – 80.5	14	285
81 - 90	80.5 – 90.5	10	295
91 - 100	90.5 – 100.5	5	300
		N=300	

From the table, we write the following:

$\frac{N}{2} = \frac{300}{2} = 150$. This indicates that the median is somewhere in the 150^{th} position when all the marks are arranged from lowest to highest.

The median class is 41 – 50 and its lower class boundary is 40.5, so L_m = 40.5.

$cf_m = 145$, $f_m = 55$, and $i = 10$.

$$\text{Median} = L_m + \left[\frac{\frac{N}{2} - cf_m}{f_m}\right] i$$

$$\text{Median} = 40.5 + \left[\frac{150 - 145}{55}\right] 10$$

Median = 41.41

Class Activity 2

1) A manufacturer of electronic components is interested in determining the lifetime of a certain type of battery. A sample, in hours of life, is as follows: 123, 116, 122, 110, 175, 126, 125, 111, 118, and 117. Find the median of this set.

Solution:

2) Thirty automobiles were tested for fuel efficiency, in miles per gallon (*mpg*). The following frequency distribution was obtained. Find the median.

Class Boundaries	Frequency	
7.5-12.5	3	
12.5-17.5	5	
17.5-22.5	15	
22.5-27.5	5	
27.5-32.5	2	

Solution:

3) Mode

Mode is the value/element in a set that appears most frequently and the **modal class** is the class with the highest frequency.

Note: If there are two elements with the same highest frequency then the distribution is said to be **bi-modal.**

A) Mode of Ungrouped Data

Example 1: What is the mode the following distribution?

1, 3, 3, 4, 4, 4, 4, 4, 5, 6, 7, 7

Solution: The Mode is 4 since it's the one that appears most.

B) Mode of Grouped Data

Procedure:

Step 1) Find a modal class = class with highest frequency

Step 2) Mode = $L_{mo} + \left[\frac{\Delta_1}{\Delta_1 + \Delta_2}\right] i,$

where

L_{mo} = the lower class boundary of the modal class

Δ_1 = the difference between the frequency of the modal class and the frequency of the class **before** the modal class.

Δ_2 = the difference between the frequency of the modal class and the frequency of the class **after** the modal class.

i = class width or class size

Example 1: Find the mode of the given data below:

Time to travel to work in minutes	Frequency
1 - 10	8
11 - 20	14
21 - 30	12
31 – 40	9
41 - 50	7

Solution: Highest frequency = 14 so the modal class is 11 – 20.

$L_{mo} = 10.5$; $\Delta_1 = 14 - 8 = 6$

$\Delta_2 = 14 - 12 = 2$; $i = 10$

Mode = $L_{mo} + \left[\frac{\Delta_1}{\Delta_1 + \Delta_2}\right] i$

Mode = $10.5 + \left[\frac{6}{6+2}\right] 10 = 18$

Class Activity 3

1) Find the mode of the data:
59, 65, 61, 62, 53, 55, 70, 60, 64, 56, 58, 58, 62, 62, 68, 65, 56, 59, 68, 61, and 67.

Answer:

2) In the following frequency distribution, what is the modal class?

Measure of gain	Number of transistors
83.5–85.5	6
86.5–88.5	39
89.5–91.5	27
92.5–94.5	15
95.5–97.5	3

Answer :

3) Find the mode of the given distribution:

Seconds	**Frequency**
51 - 55	2
56 - 60	7
61 - 65	8
66 - 70	4

Solution:

3.4 MEASURES OF DISPERSION OR VARIATION

Dispersion is the tendency of data to be scattered over a certain interval or away from the center. It is also called spread or variation.

Measures of dispersion are useful in comparing two or more distributions in respect to disparities. A greater degree of dispersion means lack of uniformity or homogeneity in the data while a low degree of dispersion stands for uniformity or homogeneity. The most common measures of dispersion are:

1. Range
2. Variance
3. Standard deviation

1) Range

The range is the difference between the largest and the smallest values.

Range = Largest Value – Smallest Value

For **example,** if the lowest and highest marks scored in a test are 15 and 95,

then the range = 95 -15 = 80.

Even though the range is the easiest measure of dispersion to calculate, it is not considered a good measure of dispersion as it does not utilize the other information related to the spread. The outliers, either the extremely low value or extremely high value can affect the range considerably.

Class Activity 1

1) The range for the following data 126, 128, 130, 132, 131, 143, 134, 135, 134, 134 is……

(a) 134
(b) 17
(c) 27

2) Variance

The **variance** is the average of the squared differences from the mean. It is a measure of variation that considers the position of each observation relative to the mean.

A) Variance of **Ungrouped Data** is given by:
Sample Variance

$$s^2 = \frac{\sum(x_i - \bar{x})^2}{n-1} \quad Or \quad \frac{n\sum x_i{}^2 - (\sum x_i)^2}{n(n-1)}$$

B) Variance of **Grouped Data** is given by:
Sample Variance

$$\boldsymbol{s^2 = \frac{\sum f_i(x_i - \bar{x})^2}{n-1}} \quad Or \quad \frac{n\sum f_i x_i{}^2 - (\sum f_i x_i)^2}{n(n-1)}$$

3) Standard Deviation

Standard Deviation is the positive square root of the variance.

$$\boldsymbol{s = \sqrt{s^2} = \sqrt{Variance}}$$

Example 1: Find the variance and standard deviation of the data from a sample of 11 observations:

1, 3, 3, 4, 4, 4, 4, 5, 6, 7, 7

Solution:

Step 1) Find the **mean**.

$$\text{Mean} = \frac{1+3+3+4+4+4+4+5+6+7+7}{11}$$

$$\text{Mean} = \frac{48}{11} = 4.36$$

Step2) Find the squared differences.

x	$x - \bar{x}$	$(x - \bar{x})^2$
1	-3.36	11.29
3	-1.36	1.85
3	-1.36	1.85
4	-0.36	0.13
4	-0.36	0.13
4	-0.36	0.13
4	-0.36	0.13
5	0.64	0.41
6	1.64	2.69
7	2.64	6.97
7	2.64	6.97
		$\sum(x - \bar{x})^2$ = 32.55

Step 3) Calculate the sample variance.

$$s^2 = \frac{\sum(x_i - \bar{x})^2}{n-1} = \frac{32.55}{11-1} = 3.26$$

Step 4) Calculate the sample standard deviation.

$$s = \sqrt{s^2} = \sqrt{3.26} = 1.81$$

Example 2. Find the sample variance and standard deviation of the data:

Seconds	Frequency
51 - 55	2
56 - 60	7
61 - 65	8
66 - 70	4
	n = 21

Solution:

Step 1) Find the mean.

.Seconds	f	x_i	fx_i
51 - 55	2	53	106
56 - 60	7	58	406
61 - 65	8	63	504
66 - 70	4	68	272
	n = 21		1288

$$\bar{x} = \frac{\sum f_i x_i}{n} = \frac{1288}{21} = 61.33$$

Step 2) Complete the table based on the formula for variance and standard deviation.

Seconds	f	x_i	$(x_i - \bar{x})^2$	$f(x_i - \bar{x})^2$
51 - 55	2	53	69.39	138.78
56 - 60	7	58	11.09	77.63
61 - 65	8	63	2.79	22.32
66 - 70	4	68	44.49	177.96
	n = 21			$\sum = 416.69$

Step 3) Calculate the sample variance.

$$s^2 = \frac{\sum f_i(x_i - \bar{x})^2}{n-1} = \frac{416.69}{21-1} = 20.83$$

Step 4) Calculate the sample standard deviation.

$$s = \sqrt{s^2} = \sqrt{20.83} = 4.5$$

Class Activity 2

1) The IQ's (Intelligence Quotient) of 5 students are 108, 112, 127, 118, and 113. Determine the following: i) Variance, and ii) Standard deviation

Solution:

2) Given the frequency distribution of a sample of 50 pieces of luggage inspected randomly in an airport. Determine the following: i) Variance, and ii) Standard Deviation,

Weight (*kgs*)	Number of Pieces
7 - 9	2
10 - 12	8
13 - 15	14
16 - 18	19
19 - 21	7
	n = 50

Solution:

WORKSHEET 3

Section-A

I) Circle the correct answer in the following questions.

1) The science of collecting, summarizing, and analyzing numerical data is...........

(a) Data
(b) Population
(c) Statistics

2) .... is a set of elements or observations selected at random to draw some conclusions about the population.

(a) Survey
(b) Sample
(c) Statistics

3) .... refers to number of observations chosen randomly in order to investigate the population.

(a) Statistics
(b) Sample size
(c) Data

4) The heights of students measured in centimeters are classified as...........

(a) Qualitative data
(b) Discrete data
(c) Continuous data

5) Number of chairs in a classroom is an example of ....data.

(a) Quantitative-continuous
(b) Qualitative
(c) Quantitative-discrete

6) If the class boundaries are 6.45 - 9.55, then the class interval is ....

(a) 6.5 – 9.5
(b) 7.0 - 9.0
(c) 6.0 – 9.0

7) The graph between class boundaries and frequency is called ....

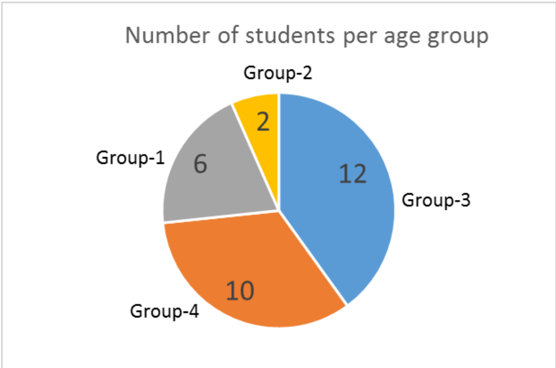
(a) Histogram
(b) Pie chart
(c) Bar chart

8) The class size of the given distribution is

Seconds	Frequency
61 - 65	5
66 - 70	10
71 - 75	11
76 - 80	7

(a) 8
(b) 4
(c) 5

9) In the following pie chart what percentage (%) is the age of group -3?



(a) 40 %
(b) 20 %
(c) 12 %

10) Which of the following is measure of central tendency of data?

(a) Range
(b) Mode
(c) Variance

11) Which of the following is measure of dispersion of data?

(a) Mean
(b) Range
(c) Median

12) The range for the following data 26, 28, 30, 32, 31, 43, 34, 35, 34, 34 is ………

(a) 34
(b) 17
(c) 327

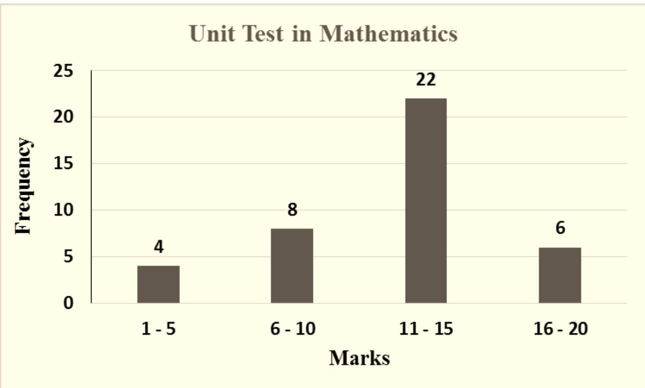
13) The frequency of ‘4’ in the following data 0, 2, 1, 3, 4, 5, 4, 4, 5 is ………

(a) 2
(b) 28
(c) 3

14) If the standard deviation of a data is 4 then the measure of its variance is …….

(a) 16
(b) 4
(c) 12

II. Given below is the bar chart for the marks of 40 students in their Unit Test in Mathematics:



i) What is the class size of the distribution?

Answer: _________

ii) Which class interval has the highest frequency?

Answer: _________

iii) What is the cumulative frequency for the marks less than 10.5?

Answer: ____________

iv) What is the relative frequency of the class 11 – 15?

Answer: __________

Section-B

Show your solution step by step in the following questions. Round off the answer to 2 decimal places.

1) An insurance company researcher conducted a survey on the number of car thefts in a large city for a period of 30 days last summer. Plot a histogram for the data using 6 classes.

52	62	51	50	69	58	77	66	53	57
75	56	55	67	73	79	59	68	65	72
57	51	63	69	75	65	53	78	66	55

Solution:

2) The number of building permits issued last year to 12 construction companies were 4, 7, 1, 7, 11, 4, 1, 15, 3, 5, 8, and 7. Treating the data as a sample, calculate the following:

i) the mean

ii) the median

iii) the mode

Solution:

3) The reaction time for a random sample of 9 patients to a stimulant were recorded as 2.5, 3.6, 3.1, 4.3, 2.9, 2.3, 2.6, 4.1, and 3.4 seconds. Find the following:

i) the range

ii) the variance

iii) the standard deviation

Solution:

4) The table below shows the marks of 40 sample students who recently took a Unit Test in Mathematics:

Unit Test Marks	Frequency
1 - 5	4
6 - 10	8
11 - 15	22
16 - 20	6

Calculate the average mark of the class.

Solution:

5) The distribution below gives the weight of 30 students of a class. Find the median class and median weight of the students.

Weight (in Kg)	Number of students
40 - 45	2
45 - 50	3
50 - 55	8
55 - 60	6
60 - 65	6
65 - 70	3
70 - 75	2

Solution: (Answers: 55 – 60 and 56.67 kg)

6) The following table shows the ages of the patients admitted in a hospital during a year:

Age (in Years)	Number of patients
5 - 15	6
15 - 25	11
25 - 35	21
35 - 45	23
45 - 55	14
55 - 65	5

Find the modal class and mode of the data given above.

Solution: (Answers: 35 – 45 and 36.8 Years)

7) The distribution below represents the net worth (in millions of dollars) of 45 national corporations. Find the standard deviation for the following:

Class Intervals	Frequency
20-30	3
31-41	10
42-52	7
53-63	15
64-74	8
75-85	2

Solution:

(UNIT-4) PROBABILITY

4.1 BASIC CONCEPTS

1) Experiment: Any process that generates a set of data or outcomes is called experiment.

2) Sample Space: The set of all possible outcomes of any experiment is called sample space and is generally denoted by *S*. The possible outcomes are called sample point.

Example :

Experiment	Sample space
1) Throwing/tossing a fair die 	$S = \{1, 2, 3, 4, 5, 6\}$
1) Tossing of a coin	$S = \{Head, Tail\}$
2) Tossing of two coins	$S = \{HH, HT.TH, TT\}$
3) A family of two children	$S = \{BB, BG, GB, GG\}$

3) Event: An event is a subset of the sample space.

Example 1: Let A = the event that an **even number** turns up when a die is tossed once.

$$A = \{ 2, 4, 6\} \ or \ A = \{ even\}$$

Example 2: Let ***A*** = the event of getting a number greater than 6 in a toss of a single die. In this case $A = \{ \quad \} = \emptyset$.

Class Activity 1

1) Find the sample space for rolling a pair of dice once.

4) Counting the Sample Points

In predicting the chance of any event to likely happen, we need to know how many ways it would happen out of the total possibilities in the sample space.

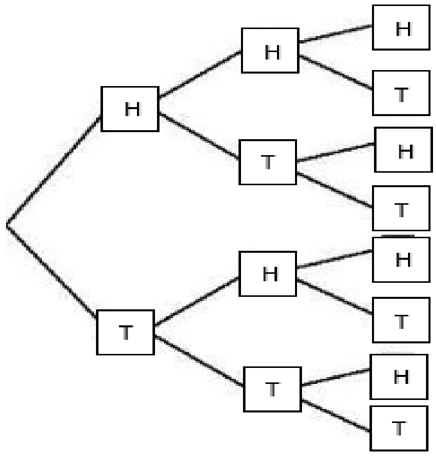
Some methods of listing and counting the elements of sample space are discussed below:

4.1) Tree Diagram Method

Example 1: If a coin is flipped 3 times, list all the possible outcomes using tree diagram method.

Solution: Let H = Head and T = Tail

Following is the Tree diagram:



Sample space = $S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

There are 8 possible outcomes, that is, n(S) = 8

<u>Class Activity 2</u>

1) Suppose that 3 items are selected at random from a manufacturing process. Each item is inspected and classified as D for defective and N for non-defective. Show the tree diagram for the possible outcomes and list the elements of the sample space.
Solution:

2) Write the sample space of 3 children in the family. Use 'B' for boys and 'G' for girls.

4.2) Generalized multiplication Rule

If an operation can be performed in n_1 ways, if for each of these a second operation can be performed in n_2 ways, if for each of these a third operation can be performed in n_3 ways, and so on, then the sequence of k operations can be performed in $n_1 \cdot n_2 \cdot n_3 \cdot \ldots \cdot n_k$ ways.

Example 1: How many lunches are possible consisting of soup, a sandwich, dessert, and a drink if one can select from 4 soups, 3 kinds of sandwiches, 5 desserts, and 4 drinks?

Solution: The number of lunches would be:

(4)(3)(5)(4) = 240.

Example 2: Find the number of outcomes in rolling a pair of dice.

Solution: Number of outcomes in a die is 6 so the number of outcomes in pair of dice is obtained by: $(6)(6) = 6^2 = 36$.

Example 3: Find the number of outcomes in flipping a coin three times.

Solution: Number of outcomes in flipping a coin is 2 so the number of outcomes in flipping a coin 3 times is given by: $(2)(2)(2) = 8$.

This can be obtained using the power rule: 2^n where the base 2 shows the number of outcomes for each trial and *n* represents the number of trials.

<u>Class Activity 3</u>

1) In how many ways can a true-false test consisting of 3 questions be answered?

4.3) Permutation: A permutation is an arrangement of a set of distinct objects considering the order they are to be arranged.

The following rules are used in permutation:

Rule 1: ***The number of permutations of n distinct objects is n!***

$$\boldsymbol{n! = n(n-1)(n-2)\ldots 1!}$$

In mathematics, $\boldsymbol{n}!$ means the **factorial** of a non-negative integer **n**. It is the product of all positive integers less than or equal to **n**.

Example 1: 5! = 5 × 4 × 3 × 2 × 1 = 120.

Note that 1! = 1 and 0! = 1.

Example 2: How many ways that four students can be seated in a row with 4 consecutive seats available?

Solution: Since $n = 4$, then $\boldsymbol{n}! = \boldsymbol{4}!$

$\boldsymbol{n}! = \boldsymbol{4}! = (\boldsymbol{4})(\boldsymbol{3})(\boldsymbol{2})(\boldsymbol{1}) = \boldsymbol{24}$ ways**.**

Rule 2: ***The number of permutations of n distinct objects taken r at a time is*** $_nP_r$

Example 1: Two lottery tickets are drawn from 20 for first and second prizes. Find the number of sample points in the sample space *S*.

Solution: The total number of sample points is

$_nP_r = 380$

Rule 3: ***The number of permutations of n distinct objects arranged in a circle is (n – 1)!***

Example 1: Five people are to seat at a round table for a meeting. How many ways they can be arranged?

Solution: The number of possible arrangements is:

(n – 1)! = (5 – 1) ! = 4! = 24

4.4) Combinations: These are selections of ***r*** objects from ***n*** without regard to order. A combination creates a partition of two (2) cells, one cell containing the ***r*** objects selected and the other cell containing the **(*n* – *r*)** objects that are left.

Rule*: *The number of combinations of n distinct objects taken r at a time is $_nC_r$ ***:***

Example 1: How many ways are there to select 3 candidates from 8 equally qualified recent graduates for a job in an accounting firm?

Solution: The number of combinations is

$_nC_r = 56$

Class Activity

A) Circle the correct answer in the following questions.

1) ________ is an arrangement of distinct objects in specific order.
(a) Permutation
(b) Combination
(c) Frequency distribution

2) ________ is selection of distinct objects.
(a) Permutation
(b) Combination
(c) Frequency distribution

3) The number of ways of selecting 3 boys from 5 boys is _________
(a) 120
(b) 10
(c) 20

4) The number of ways that 4 people can sit at a round table is _________
(a) 24
(b) 6
(c) 10

5) The number of 2 letter words that can be made with the letters of the word 'BASIC' is _______
(a) 120
(b) 10
(c) 20

B) Show your solution step by step in the following questions.

1) In how many ways five students can be seated in a row with 5 consecutive seats available?
Solution:

2) How many 3 digit numbers can be made using the digits 2, 3 and 7 without repetitions?
Solution:

3) How many 2 digit numbers can be made using the digits 2, 4, 5 and 9 without repeating the digits?
Solution:

4) i) How many distinct permutations can be made with the letters of the word “COLUMNS”?
ii) How many of these permutations start with the letter “C”?
iii) How many of these permutations start with the letter “S” and end with “C”?
Solution:

5) A committee of 2 boys and 3 girls is to be formed from a group of 4 boys and 6 girls. How many different committees can be formed?

Solution:

6) A box contains 3 red, 4 white and 5 blue balls. How many selections of 3 balls can be made so that
i) All the three balls are white
ii) Two white balls and one blue ball
iii) There is one ball of each color
Solution:

4.2 PROBABILITY

Definition of Probability

Probability is the extent to which something is going to happen.

The ***probability*** of an event is the measure of the **chance** that the event will occur as a result of an experiment. The ***probability*** of an event ***A*** is the number of ways event ***A*** can occur divided by the total number of possible outcomes.

Rule:. *If an experiment can result in any one of N different equally likely outcomes, and if exactly n of these outcomes corresponds to event A, then the probability of event A is:*

$$P(A) = \frac{n}{N}$$

Note: $0 \leq P(A) \leq 1$ and $P(S) = 1$.

The probability of an event which absolutely is not going to happen (impossible to happen) is 0, or $P(\emptyset) = 0$. For example, the probability of getting a blue ball from a box containing only red and green balls is 0.

The probability of an event which absolutely (certainly) is going to happen is 1. For example, the probability of getting a blue ball from a box containing only blue balls is 1.

Example 1: A coin is tossed twice. What is the probability that 2 heads occur?
Solution:
Sample Space: $S = \{HH, HT, TH, TT\}$

Let A = event that 2 heads occur

$$P(A) = \frac{1}{4} = 0.25 \text{ or } 25\%$$

Example 2: If a card is drawn from an ordinary deck of cards, find the probability that it is a heart.
Solution: The number of possible outcomes is 52, of which 13 are hearts. If A = event that the card is drawn is the heart, then $P(A) = \frac{13}{52} = \frac{1}{4}$.

Structure of pack/deck of cards

Note: Spades and clubs are of black colour whereas Hearts and Diamonds are of red colour

Example 3: A box contains 3 red balls and 5 green balls. If one ball is drawn at random, what is the probability that it is green?

Solution: Let G = event that a ball drawn is green.

$$P(G) = \frac{5}{8}$$

Example 4: Given the frequency distribution of a sample of 50 pieces of luggage inspected randomly in an airport.

Weight (kilograms)	Number of Pieces
7 - 9	2
10 - 12	8
13 - 15	14
16 - 18	19
19 - 21	7
	n = 50

If one luggage will be picked at random, what is the probability that its weight is 13 to 15 kilograms?

Solution: Let A = the event of picking a luggage whose weight is 13 to 15 kilograms.

$$P(A) = \frac{14}{50} = \frac{7}{25}$$

Example 5: Let A = the event of drawing 1 green ball from a box containing 5 red and 2 blue balls. What is $P(A)$?

Solution: Since there is no green ball inside the box, then $P(A) = 0$.

Class Activity 1

A) Circle the correct answer in the following questions.

1) A coin is tossed two times. The probability of getting one head and one tail is

(a) $\frac{1}{4}$
(b) $\frac{1}{2}$
(c) $\frac{3}{4}$

2) A die is thrown once. The probability of getting multiple of 3 is

(a) $\frac{1}{6}$
(b) $\frac{1}{3}$
(c) $\frac{1}{2}$

3) A box contains 4 red balls and 6 green balls. If one ball is drawn at random, what is the probability that it is red?

(a) $\frac{2}{5}$
(b) $\frac{3}{5}$
(c) $\frac{4}{5}$

4) A die is thrown twice. The probability of getting the even number on both dice is

(a) $\frac{1}{2}$
(b) $\frac{1}{3}$
(c) $\frac{1}{4}$

B) Show your solution step by step in the following questions.

1) If a card is drawn from an ordinary deck of cards, find the probability that it is
i) an ace
ii) spade
iii) 6 of club
Solution:

2) A die is thrown once. What is the probability of getting a prime number?
Solution:

3) A coin is tossed two times. Find the probability of getting
i) both tails.
ii) at least one head.
Solution:

4) A die is thrown twice. What is the probability of getting the same number on both of them?
Solution:

5) A die is thrown twice. What is the probability of getting the sum 6?
Solution:

6) A bag contains cards with the numbers:
1, 2, 3,………..,50.
One card is drawn at random from the bag. What is the probability of getting a perfect square?
Solution:

7) A box contains 2 red balls, 3 white and 4 black balls. If 3 balls are drawn at the same time, what is the probability that

i) 2 are white balls and 1 is red?

ii) all 3 are black balls?

iii) all 3 are different colors?

Solution: Hint: Use combinations.

4.3 RULES OF PROBABILITY

Simple and Compound Events

If an event is a set containing only one element of the sample space, then it is called a **simple event**. A **compound event** is one that can be expressed as the union of simple events.

Example 1: In drawing a card from a deck of 52 playing cards, if we are interested in the ***suit*** of the card, then the sample space would be:

$S = \{heart, spade, diamond, club\}.$

The subset $\boldsymbol{A} = \{\boldsymbol{heart}\}$ is a simple event. Now the event ***B*** of drawing a red card is a compound event because there are two possible red suits since $B = \{heart \ \cup diamond\}$ or

$B = \{heart, diamond\}.$

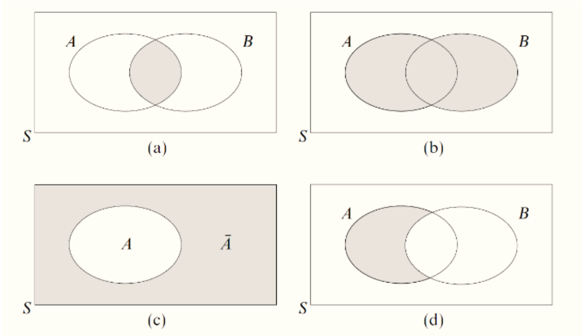
Class Activity 1

1) Write S in column 2 if the event in column 1 is a simple event and write C if the event is a compound event.

Column 1 (Event)	Column 2 (S / C)
i) Getting an even number in a single toss of a die.	C
ii) 3 boys out of 3 children in a family.	
iii) 2 boys and 1 girl out of 3 children in a family.	
iv) Getting at least one H in tossing a pair of coins.	
v) Drawing a "4" of spades in a deck of cards.	

Venn diagrams

A common graphical representation of the outcomes of an experiment is the use of Venn diagrams



The shaded regions shows:

(a) $A \cap B$, the intersection of two events A and B,

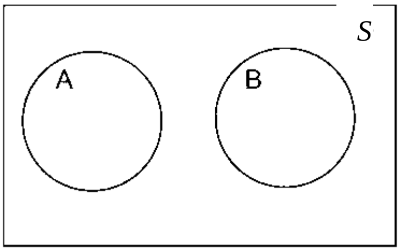
(b) $A \cup B$, the union of events A and B,

(c)the complement $\overline{A}$ of an event A,

(d) $A \cap \overline{B}$, those outcomes in A that do not belong to B.

1) *Additive Rule:*

Mutually Exclusive Events: Two events A and B are mutually exclusive if $A \cap B = \emptyset$, meaning the two events cannot occur at the same time



Example : Let A = {2} , B = {3} and

S = { 1, 2, 3, 4, 5, 6 }

Exhaustive events: Two events A and B are said to be collectively exhaustive if

A U B = S= Sample space

Example 1: Let A = {2, 4, 6} , B = {1, 2, 3, 4, 5}

and S = { 1, 2, 3, 4, 5, 6 } then A U B = S

Hence A and B are collectively exhaustive events.

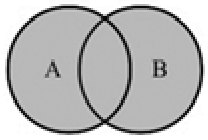
Example 2: Let A = {2, 4, 6} , B = {1, 3, 5}

and S = { 1, 2, 3, 4, 5, 6 } then A U B = S

Here A and B are mutually exclusive and also collectively exhaustive events.

<u>Addition Rule of Probability</u>

(i) *If A and B are any two events, then*

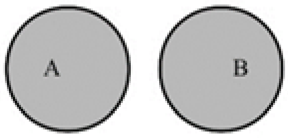


A∪B

Case 1

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

(ii) *If A and B are two mutually exclusive events, then*



A∪B

Case 2

$$P(A \cup B) = P(A) + P(B)$$

Generalized Rule:

If $A_1, A_2, A_3, \ldots, A_n$ are mutually exclusive, then

$P(A_1 \cup A_2 \cup \ldots \cup A_n) = P(A_1) + P(A_2) + \cdots + P(A_n)$

Note: If $A_1, A_2, A_3, \ldots, A_n$ are mutually exclusive and collectively exhaustive, then

$P(A_1 \cup A_2 \cup \ldots \cup A_n)$

$= P(A_1) + P(A_2) + \cdots + P(A_n) = P(S) = 1$

Example 1: One student is enrolled in mathematics and English courses. The probability that he passes mathematics is $\frac{2}{3}$ and the probability that he passes English is $\frac{4}{9}$. If the probability that he passes at least one course is $\frac{4}{5}$, what is the probability that he passes both courses?

Solution: Let M = event of "passing mathematics" and E = event of "passing English".

The event of passing at least one course is $M \cup E$ and the event of passing both courses is $M \cap E$. Therefore, $(M) = \frac{2}{3}$, $P(E) = \frac{4}{9}$, and

$P(M \cup E) = \frac{4}{5}$.

By using the Additive Rule (i), we write:

$$P(M \cup E) = P(M) + P(E) - P(M \cap E)$$

$$P(M \cap E) = P(M) + P(E) - P(M \cup E)$$

$$P(M \cap E) = \frac{2}{3} + \frac{4}{9} - \frac{4}{5} = \frac{14}{45}$$

Example 2: Three candidates are seeking a job. Candidates 'A' and 'B' are given the same chances of getting the job. Candidate 'C' is given twice the chance of 'B'.

a) What is the probability that *C* wins?

b) What is the probability that either A or B wins?

Solution:

a) If we let $P(A) = x$ and $P(B) = x$, then $P(C) = 2x$.

By using Additive Rule (iii):

$$P(A) + P(B) + P(C) = 1$$

$$x + x + 2x = 1$$

$$4x = 1, \quad x = \frac{1}{4}$$

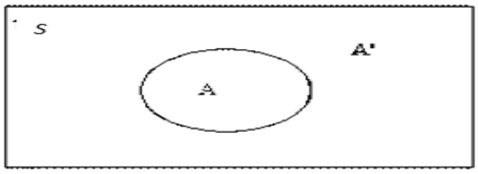
so $P(C) = 2x = 2\left(\frac{1}{4}\right) = \frac{1}{2}$

b) The event that either *A* or *B* wins is denoted by $A \cup B$. By using Additive Rule (ii), we write:

$$P(A \cup B) = P(A) + P(B)$$

$$P(A \cup B) = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

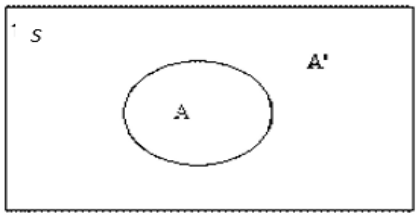
2) Complement of an Event: The complement of an event *A* with respect to the sample space *S* is the set of elements of *S* that are not in *A*. We denote the complement of event A by the symbol A' or $\overline{A}$.



Example : Define $S = \{1,2\ 3,4\ 5,6\}$ and ***A*** is the event that an odd number will turn up on a single die, then we write: $A = \{1, 3, 5\}$ and $A' = \{2, 4, 6\}$.

If A and A′ are complementary events, then

$P(A) + P(A') = 1$



Example 1: In a batch of 45 lamps there are 10 faulty lamps. If one lamp is drawn at random, find the probability of it being

i) faulty and

ii) satisfactory

Solution: i) If F represents the event of picking a faulty lamp, then $P(F) = \frac{10}{45} = \frac{2}{9}$

ii) The event of picking a satisfactory lamp is therefore $\overline{F}$, so $P(\overline{F}) = 1 - \frac{2}{9} = \frac{7}{9}$

Example 2: What is the probability that there will be a least one boy out of the three children in a family?

Solution: Let *E* = event of having at least 1 boy and *E'* be the event of having no boy at all (all 3 children are girls). We define the sample space $S = \{BBB, BBG, BGB, BGG, GBB, GBG, GGB, GGG\}$.

This means that there are 8 ways of classifying the genders of the 3 children.

Note: By inspection of the elements of set *S*, we can write:

$$E' = \{GGG\} \text{ and } P(E') = \frac{1}{8}$$

The probability of having **at least one boy** is:

$$P(E) = 1 - P(E') = 1 - \frac{1}{8} = \frac{7}{8}$$

Note: The question can also be solved directly from Sample space.

Class Activity 2

1) If $(A) = \frac{7}{20}, P(B) = \frac{1}{4}$ and $P(A \cap B) = \frac{2}{5}$, then find $P(A \cup B)$.

Solution:

2) If A and B are mutually exclusive events and $P(A \cup B) = \frac{7}{10}, P(B) = \frac{1}{4}$, then find $P(A)$.

Solution:

3) If $P(A) = \frac{3}{8}$ find $P(A')$

Solution:

4) If $P(A) = \frac{11}{24}, P(B) = \frac{3}{8}$ and $P(A \cup B) = \frac{1}{12}$, then find $P(A \cap B)'$.

Solution:

5) If *A*, *B*, and *C* are mutually exclusive events and P(*A*) = 0.2, P(*B*) = 0.3, and P(*C*) = 0.4, find:

i) $P(A \cup B \cup C)$ and

ii) $P(C')$

Solution:

6) The probability for a contractor to get a road contract is $\frac{2}{3}$ and to get a building contract is $\frac{5}{9}$. The probability to get at least one contract is $\frac{4}{5}$. Find the probability of getting both contracts.

Solution:

7) In a class of 24 students, 15 are taking mathematics, 12 are taking Physics, and 7 are taking both mathematics and Physics. If one student is selected at random, what is the probability that:

i) the student is taking mathematics but not Physics?
ii) the student is taking mathematics or Physics?
iii) the student is taking neither of these subjects?
Solution: Hint / Idea to solve: Use Venn diagram

3) Multiplicative Rule:

Independent Events

Two events, *A* and *B*, are **independent** if the outcome of the first event does not influence the outcome of the second event.

Example : If you flip a coin and it lands on tails and flip it again and it lands on heads, neither outcome influences the other.

If two events A and B are independent, then

$$P\,(A\,\cap B) = P(A).P(B)$$

In other words, if in an experiment the conditions are the same for each trial, then we are in the presence of independent events. In this case the probabilities of any two events showing favorably will simply be the simple product of the probabilities calculated with the same total number of sample points.

Generalized Rule:

If events $A_1, A_2, A_3, \dots, A_k$ are independent, then

$$P\,(A_1 \cap A_2 \cap A_3 \cap \dots \cap\ A_k) = P(A_1)\cdot P(A_2)\cdot P(A_3)\dots P(A_k)$$

Example 1: Suppose we have a fuse box containing 20 fuses, of which 5 are defective. If 2 fuses are selected at random and removed from the box in succession **with replacing the first**, what is the probability that both fuses are defective?

Solution:

Let *A* = event that the first fuse is defective.

B = event that the second fuse is defective.

$$P\,(A\,\cap B) = P(A)\cdot P(\mathrm{B})$$

$$P\,(A\,\cap B) = \frac{5}{20}\cdot\frac{5}{20} = \frac{1}{16}$$

Example 2: If a six-sided die is rolled twice and the number on which it lands is noted. What is the probability that it first lands on a prime number then a 4 on the second throw?

Solution: In this situation assuming that the die is unbiased it means in both cases each number from 1 to 6 has an equal chance of showing-up.

Let A = event of landing on prime number, $A = \{2,3,5\}$
Let B = event of landing on a 4, $B = \{4\}$

The probability of it landing on a prime number is given by $P(A) = \frac{3}{6} = \frac{1}{2}$

The probability of landing on a 4 is by $P(B) = \frac{1}{6}$

∴ The probability it first lands on a prime number then on a 4 $= \frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$

Note: In both throws the size of the sample space does not change, there are 6 possible outcomes and each outcome is equally likely. The outcomes of each throw are independent.

Example 3: A small town has one fire truck and one ambulance available for emergencies. The probability that the fire truck is available when needed is 0.98, and the probability that the ambulance is available when called is 0.92. In the event that there is a building burning and some victims are injured, find the probability that both the ambulance and the fire truck are available.

Solution: Let *A* and *B* represent the respective events that the fire truck and the ambulance are available.

$P\ (A\ \cap B) = P(A).P(B)$

$P\ (A\ \cap B) = (0.98)(0.92)$

$= 0.9016$

<u>Dependent Events</u>

Another interesting scenario is one involving a situation where each time the same experiment is performed, the size of the sample space changes.
Example: If your lunchbox contains 3 sandwiches and 2 apples, when you eat one of the items, this reduces the number of choices you have when deciding to eat a second item.

Two events, A and B, are **dependent** if the outcome of one event influences the outcome of the other.

If in an experiment, the events A and B can both occur, then

$$P\ (A\ \cap B) = P(A) \cdot P(\mathrm{B}|\mathrm{A})$$

Note: The probability of occurrence of event B when the event A has already occurred is called the **conditional probability** of B when A is given and is denoted by $\cdot\, P(\mathrm{B}|\mathrm{A})$

Generalized Rule:

(i) If in an experiment, the events $A_1, A_2, A_3, \dots, A_k$ can occur, then $P\ (A_1 \cap A_2 \cap A_3 \cap \dots \cap\ A_k) = P(A_1) \cdot P(A_2|A_1) \dots P(A_k|(A_1 \cap A_2 \cap \dots \cap\ A_{k-1})$

Example 1: Suppose we have a fuse box containing 20 fuses, of which 5 are defective. If 2 fuses are selected at random and removed from the box in succession **without replacing the first**, what is the probability that both fuses are defective?

Solution: Let *A* = event that the first fuse is defective.

B = event that the second fuse is defective.

$A\ \cap B$ = is the event that *A* occurs, and then *B* occurs after A has occurred.

$$P\ (A\ \cap B) = P(A) \cdot P(\mathrm{B}|\mathrm{A}) = \frac{5}{20} \cdot \frac{4}{19} = \frac{1}{19}$$

Example 2: In a batch of 10 computers it is known that 2 are faulty. What is the probability that if two computers are selected and tested at random, the two computers first inspected are faulty?

Solution: If F_1 is the event that the first computer tested is faulty, $P(F_1) = \frac{2}{10} = \frac{1}{5}$

And if F_2 is the event that the second computer tested is also faulty, $P(F_2) = \frac{1}{9}$,

$\therefore$ The probability that the first 2 computers chosen are both faulty is given by

$$\mathrm{P}(F_1 \text{ and } F_2) = \frac{1}{5} \times \frac{1}{9} = \frac{1}{45}$$

Note: The number of computers to be tested is not the same for the two trials. For the first computer there still 10 computers to choose from and there are 2 faulty computers to consider equally likely to be picked on. In the second case the first computer is considered as already taken off hence there now 9 computers to choose from and only 1 possible faulty computer left.

<u>Class Activity 3</u>

1) If A and B are independent events and $P(A) = 0.6, P(B) = 0.7$ then find

(i) $P(A \cap B)$ (ii) $P(A \cup B)$

2) One box contains 5 white balls and 4 black balls. If 3 balls are drawn **in succession without replacement**, what is the probability that

i) the first 2 are white balls and the third is black?

ii) all 3 are black balls?

Solution:

3) A cooler contains 20 bottles of milk (500 ml each) of which 5 bottles are spoiled. If 4 bottles are drawn **in succession with replacement**, what is the probability that

i) the first 2 are spoiled and the other 2 are good?

ii) all 4 bottles are spoiled?

Solution:

4) The probability that Student X will be graduating from MTC in five years is 0.7 and the probability that Student Y will be graduating in 5 years is 0.9. What is the probability that X or Y will graduate in five years?

Solution:

5) When testing 100 soldered joints, 4 failed during a vibration test and 5 failed due to having a high resistance. If the vibration and high resistance test are independent, determine the probability of a joint failing due to

i) vibration,
ii) high resistance,
iii) vibration and high resistance
iv) vibration or high resistance

Solution:

WORKSHEET 4

Section-A

Circle the correct answer in the following questions.

1) The number of ways of selecting 2 girls from 6 girls is

(a) 10
(b) 15
(c) 20

2) The number of ways that 6 people can sit at a round table is

(a) 720
(b) 120
(c) 60

3) The number of 4 letter words can be made with the letters in the word 'STUDY' is

(a) 120
(b) 20
(c) 10

4) A box contains 2 red balls and 3 green balls. If one ball is drawn at random, what is the probability that it is red?

(a) $\frac{2}{5}$
(b) $\frac{3}{5}$
(c) $\frac{4}{5}$

5) A die is thrown once. The probability of getting prime number is

(a) $\frac{1}{2}$
(b) $\frac{1}{3}$
(c) $\frac{1}{6}$

6) A die is thrown twice. The probability of getting the odd number on both dice is

(a) $\frac{1}{4}$
(b) $\frac{1}{3}$
(c) $\frac{1}{2}$

7) A coin is tossed three times. The probability of getting all the heads is

(a) $\frac{1}{4}$
(b) $\frac{3}{8}$
(c) $\frac{1}{8}$

Section-B

Show your solution step by step in the following questions.

1) Write the sample space of tossing 3 coins.

Solution:

2) How many 3 digit numbers can be made using the digits 1, 5 and 9 without repeating the digits?

Solution:

3) How many 2 digit numbers can be made using the digits 3, 4 and 8 without repetitions?

Solution:

4) How many 4 letter words can be made with the letters in the word 'COMPUTER'?

Solution:

5) In how many ways a committee of 2 students out of 8 students can be selected?
Solution:

6) A committee of 3 boys and 2 girls is to be formed from a group of 5 boys and 4 girls. How many different committees can be formed?
Solution:

7) i) How many distinct permutations can be made with the letters of the word "INSERT"?
ii) How many of these permutations start with the letter "S"?
iii) How many of these permutations start with the letter "S" and end with "T"?
Solution:

8) A box contains 3 black and 4 green balls. How many selections of 2 balls can be made so that
(i) Both the balls are black.
(ii) Both the balls are green.
(iii) There is one ball of each color
Solution:

9) If a card is drawn from an ordinary deck of cards, find the probability that it is a queen.
Solution:

10) If a card is drawn from an ordinary deck of cards, find the probability that it is red.
Solution:

11) A coin is tossed three times. Find the probability of getting 2 heads and 1 tail.
Solution:

12) A die is thrown twice. What is the probability of getting the sum 9?
Solution:

13) The auto mall has these cars in stock.

	SUV	Compact	Mid-Sized
Foreign	20	50	20
Domestic	65	100	45

If a car is selected at random, find the probability that it is a

1) Foreign car.

2) Foreign given that it was an SUV car.

14) A box contains 3 red, 4 white and 2 black balls. If three balls are drawn at random, what is the probability of getting all different colors.
Solution:

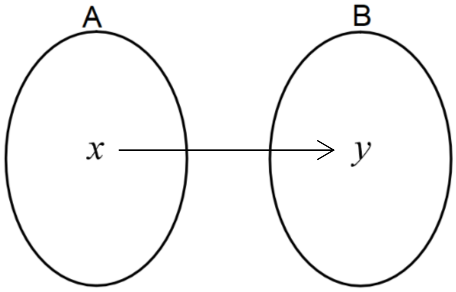
15) If one permutation is drawn randomly from among the permutations of the numbers 1, 2, 3, 4, and 5, what is the probability that it starts with an even number and no number is repeated?
Solution:

16) If A and B are independent events and $P(A) = \frac{2}{5}$ and $P(B) = \frac{3}{7}$ find $P(A'),\ P(B'),$ $P(A \cap B)$ and $P(A \cup B)$
Solution:

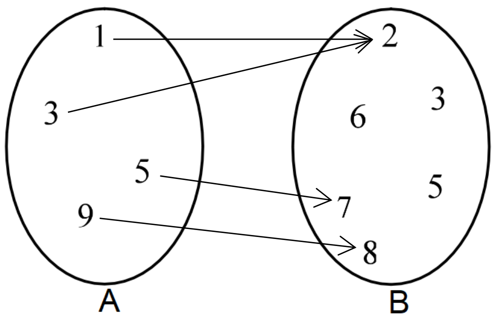
(UNIT-5) FUNCTIONS AND GRAPHS

5.1 DOMAIN, RANGE AND FUNCTION

Relation: A relation is simply a set of ordered pairs (*x*, *y*).



The **first elements** in the ordered pairs (the *x*-values), form the **domain**. The **second elements** in the ordered pairs (the *y*-values), form the **range**. Only the elements "used" by the relation constitute the range.



This mapping shows a **relation** from set A into set B This relation consists of the ordered pairs {(1,2), (3,2), (5,7), (9,8)}

- The **domain** is the set {1, 3, 5, 9}.
- The **range** is the set {2, 7, 8}.
- The **codomain** is the set.
  B = {2, 3, 5, 6, 7, 8}.
- 3, 5 and 6 are not part of the range.
- The range is the dependent variable.

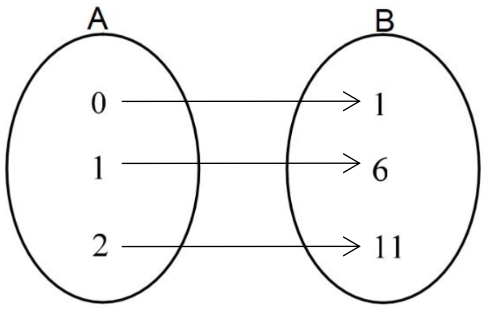
Example: Find the domain and range of the relation. $R = \{(1,1)(2,4)(3,9)\}$

Solution: Domain $= \{1,2,3\}$

Range $= \{1,4,9\}$

<u>Class Activity 1</u>

1) Find the domain and range of the relation.



Solution:

2) Find the domain and range of the relation.

x	-2	0	2
y	-8	0	8

Solution:

3) Find the *domain* and *range* of the relation.
   $R = \{(a,1)(b,2)(c,3)\}$

Solution:

Definition of a Function

A **function** is a rule that produces a correspondence between two sets of elements such that to each element in the first set there corresponds *one and only one* element in the second set.

Note:

- The set of all first elements is called **domain.**
- The set of all second elements is called **codomain.**
- The list of all elements which appears as images is called **range**.

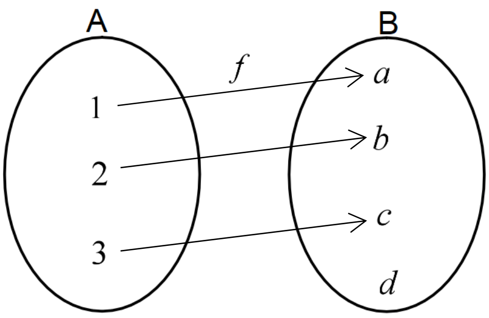
Notation of a Function

Common notations for functions are:

i) $f : A \rightarrow B$ such that the set A is the domain set and set B is the codomain.

ii) $y = f(x)$, where the set of all values of the independent variable x make up the domain set, while all values of the dependent variable y make up the codomain set or the range.

Example 1: Write domain, codomain and range from the following figure. Is '*f* 'a function?



Solution: Domain = {1, 2. 3}

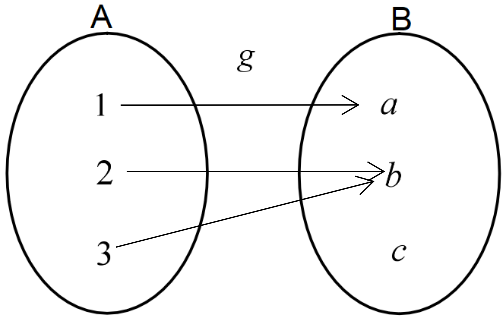
Codomain= {*a, b, c, d*}

Range = {*a, b, c*}

f (1)=*a*, *f* (2)=*b*, *f* (3)=*c*

Yes, '*f*' is a function.

Example 2: Write domain, codomain and range from the following figure. Is '*g*' a function?



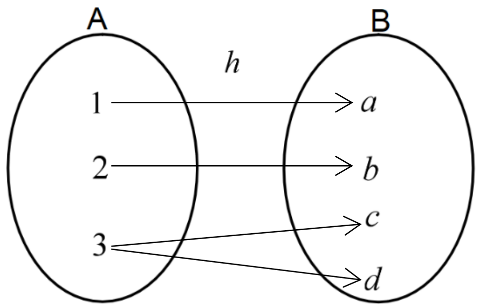
Solution: Domain = {1, 2. 3}

Codomain= {*a, b, c*}; Range = {*a, b*}

g(1)=*a*, *g*(2)=*b*, *g*(3)=*b*

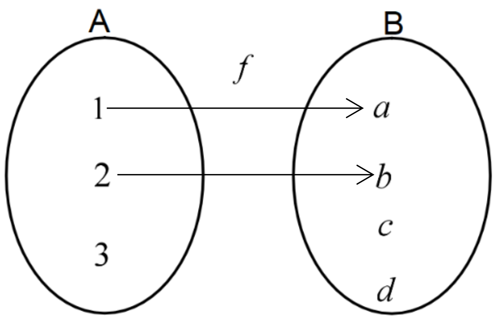
Yes, '*g*' is a function.

Example 3: Is ' *h*' a function?



Answer: No ' *h*' is not a function since element '3' in Set-A have two images.

Example 4: Is '*f* 'a function?



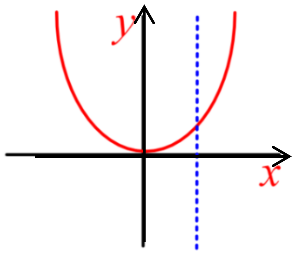
Answer: No ' *f*' is not a function since element '3' in Set-A have no image.

Determining if a Graph Defines a Function (Vertical Line Test):

A graph defines a function if each vertical line passes through exactly one point on the graph of the equation.

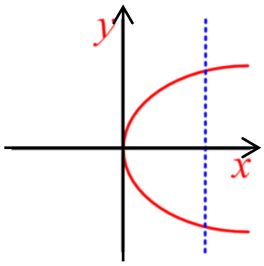
Example 5: Which of the following graphs are function?

i)



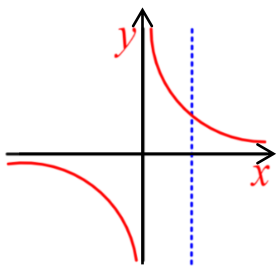
Answer: Yes, it is function

ii)



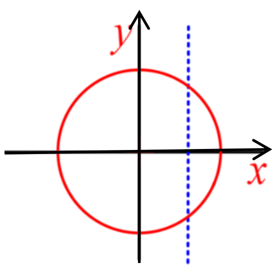
Answer: Not a function

iii)



Answer: Yes, it is function

iv)



Answer: Not a function

Class Activity 2

Circle the correct answer in the following questions.

1) Which of the following ordered pairs are function?

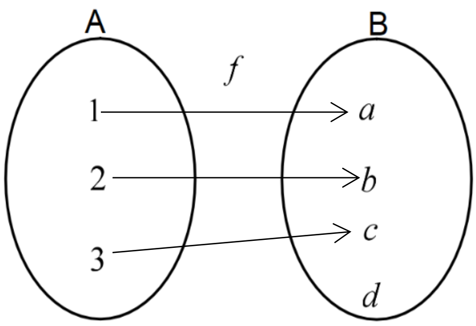
(a) $\{(b,1),(b,2),(b,3)\}$

(b) $\{(a,2),(b,2),(c,2)\}$

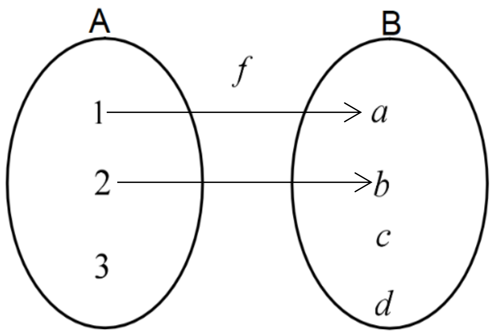
(c) $\{(1,1),(1,2),(2,3)\}$

2) Which of the following is a function?

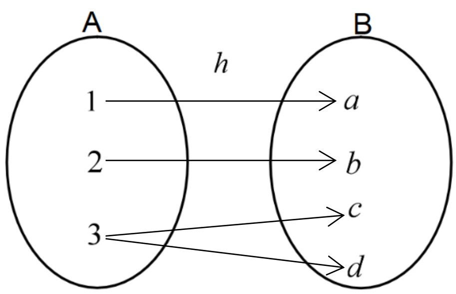
(a)



(b)

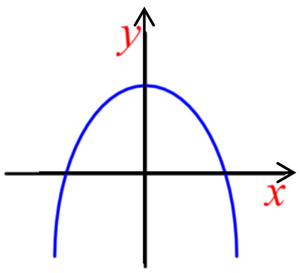


(c)

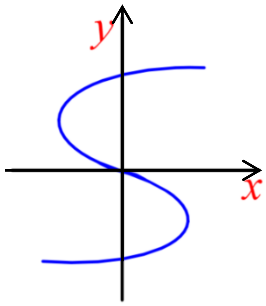


3) Which of the following is a function?

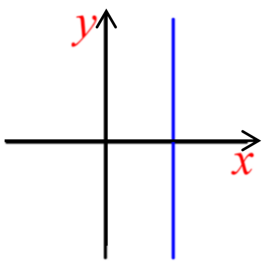
(a)



(b)



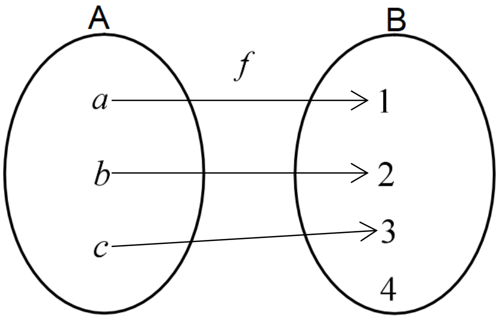
(c)



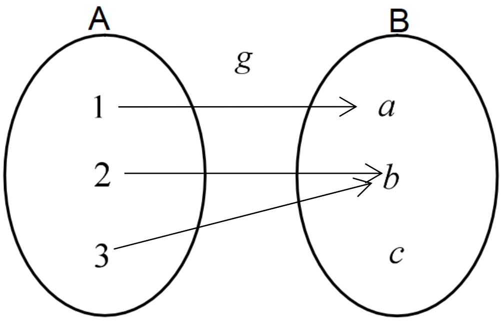
<u>One to one function</u>: A function 'f' from a set A to a set B is said to be one-to-one if no two distinct elements in A have the same image under 'f'.

Example 1:

Following is one to one function:



Following is **not one to one** function:

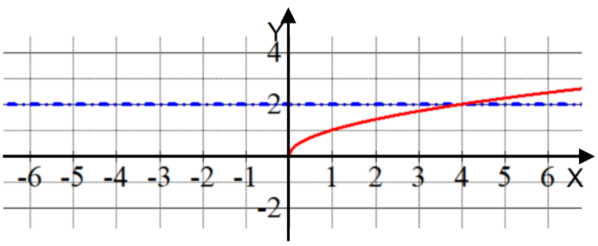


<u>Determining if a Graph Defines a One to One Function (Horizontal Line Test):</u>

A graph defines a one-to-one function if each horizontal line passes through exactly one point on the graph of the equation.

Example 2: Which of these graphs represent one to one functions?

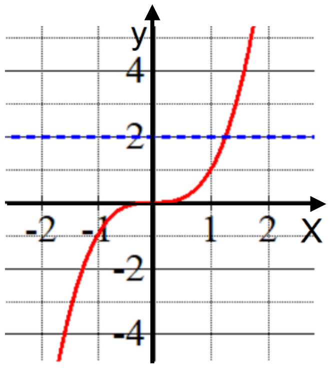
(i) Graph for $f(x) = \sqrt{x}$



Solution:

Since horizontal line cuts the graph only at single point it is one-to-one function.

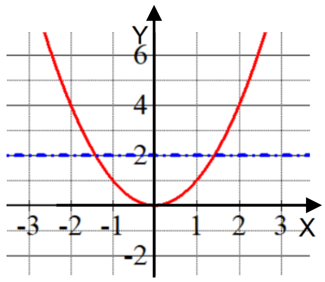
(ii) Graph for $f(x) = x^3$



Solution:

Since horizontal line cuts the graph only at single point it is one-to-one function.

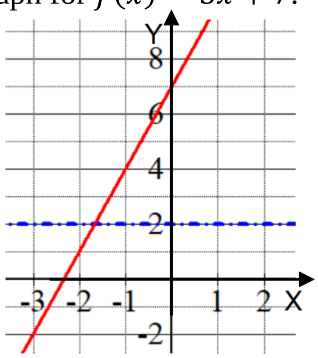
(iii) Graph for $f(x) = x^2$.



Solution:

Since horizontal line cuts the graph at two points it is not a one-to-one function.

(iv) Graph for $f(x) = 3x + 7$.



Solution:

Since horizontal line cuts the graph only at a single point, it is a one-to-one function.

Determining if a Equation Defines a One to One Function (Analytic Method):

Example 3: Which of the following are one to one functions?

(i) $f(x) = x^3$

Answer: Yes, it is one to one function.

Reason: In $f(x) = x^3$

Put $x = 1$, we get $f(1) = 1^3 = 1$

Put $x = -1$ we get $f(-1) = (-1)^3 = -1$

Since $f(-1) \neq f(1)$ so it is one to one function.

Note: 1) If all the powers of x are odd then it is one to one function.

2) Any constant has even power of x. For example: $3 = 3x^0$

(ii) $f(x) = x^2$ $or\ y = x^2$
Answer: No, it is not one to one function.

Reason: In $f(x) = x^2$

Put $x = 1$ we get $f(1) = 1^2 = 1$

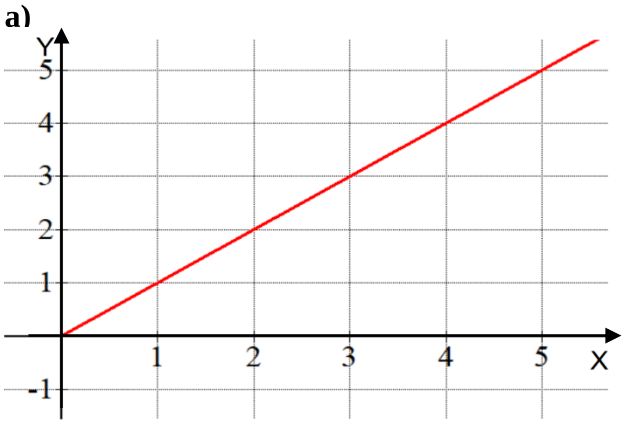
Put $x = -1$ we get $f(-1) = (-1)^2 = 1$

Since $f(-1) = f(1)$ so it is not one to one function.

Class Activity-1

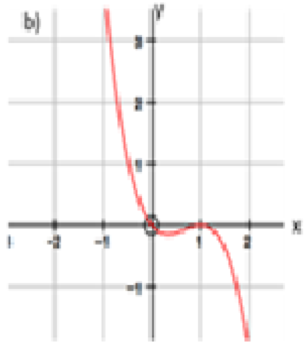
1) Which of the following graphs represent one to one functions? **(Use the horizontal line method)**

a)



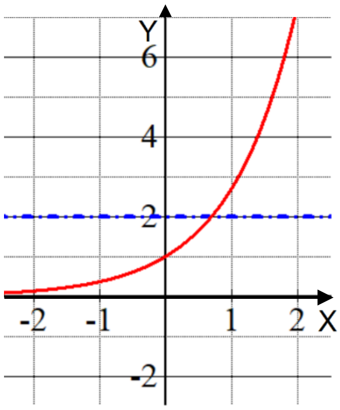
Answer:

b)



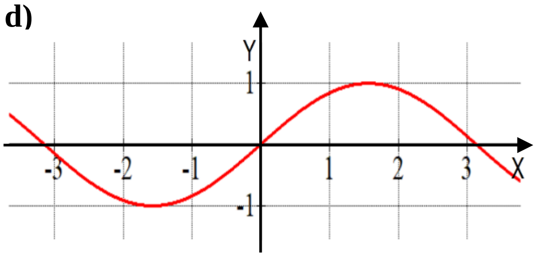
Answer:

c)



Answer:

d)



Answer:

2) Which of the following are one to one functions? (Use the analytic method)

a) $y = \frac{7x-2}{3}$

Answer:

b) $y = 3x^2 - 1$

Answer:

c) $y = x^4 - 1$

Answer:

d) $y = 2x^3 + 1$

Answer:

3) Which of the following sets represent one to one function?

a) $\{(1, a), (2, b), (3, b)\}$

Answer:

b) $\{(a, 1), (b, 2), (c, 3)\}$

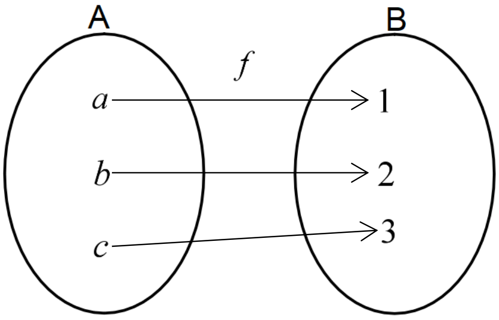
Answer:

<u>Onto function</u>: A function 'f' from a set A to a set B is said to be onto if for each element y in B there exist an element x in A such that $f(x) = y$

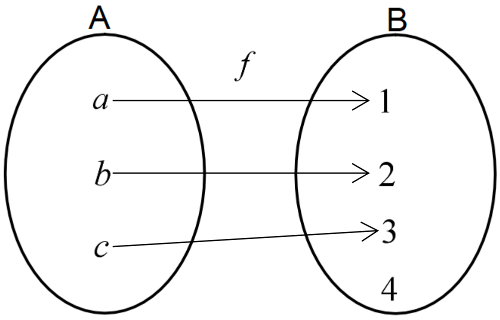
In other words a function 'f' from a set A to a set B is said to be onto if the range of f equals B, that is $f(A) = B$

Example:

Following is onto function:

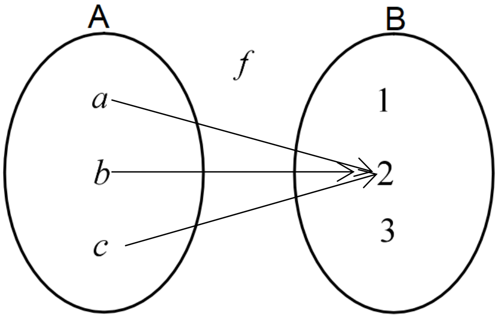


Following is **not** onto function:



<u>Constant function:</u> A function 'f' from a set A to a set B is said to be Constant function if all elements of A are mapped to single element of B.

Example 1:

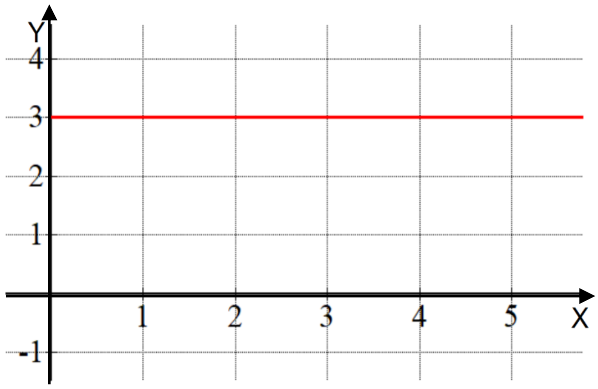


Example 2: $f(x) = 3$ where $x \in N$

Solution: Its graph is

$f(1) = 3; f(2) = 3; f(3) = 3; f(100) = 3$

so,



Applications of function:

Example: The number of computers infected by a computer virus increases according to $v(t) = t^2 + 2$, where t is the time in hours. Find

(a) The initial number of infected computers

(b) At $\boldsymbol{v(1)}$ i.e. after one hour

(c) At $\boldsymbol{v(2)}$ i.e. after two hours

Solution:

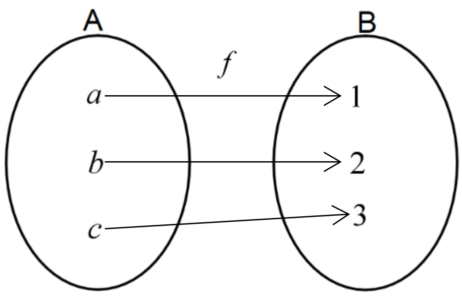
$$v(0) = 0^2 + 2 = 2$$
$$v(1) = 1^2 + 2 = 3$$
$$v(2) = 2^2 + 2 = 6$$

Class Activity-2

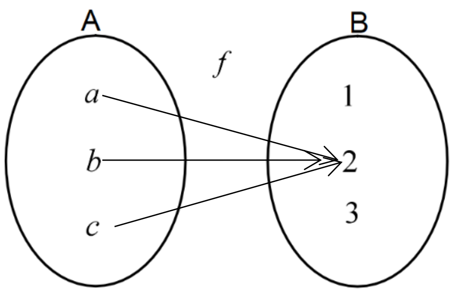
I) Circle the correct answer in the following questions.

1) Which of the following relations is onto function?

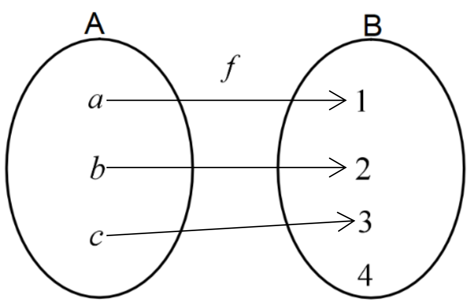
(a)



(b)



(c)



2) If $f(x) = 5$ where $x \in N$ then $f(x)$ is called ……………… function.

(a) Constant

(b) One-to-one

(c) Onto

II) The number of students in the ground is given by $f(x) = x^3 + 1$, where 'x' is time in hours.
Find number of students in the ground after 2 hours.

Solution:

5.3 INVERSE FUNCTION

<u>Inverse Function</u>: If f is a one-to-one and onto function, then the **inverse** of f, denoted by f^{-1}, is the function formed by reversing all the ordered pairs in *f*.

<u>Properties of Inverse Function</u>

If f^{-1} exists, then

1. f^{-1} is a one-to-one and onto function.

2. Domain of f^{-1} = Range of *f*

3. Range of f^{-1} = Domain of *f*

Note: If *f* is not one-to-one, then *f* **does not have an inverse** and f^{-1} **does not exist.**

Example 1: Show that the function $f(x) = 3x - 4$ **is one-to-one. Find its inverse.**

Solution:

Step 1: Verify that **f** is one-to-one.

$$f(a) = f(b) \Rightarrow 3a - 4 = 3b - 4$$
$$\Rightarrow 3a = 3b$$
$$\Rightarrow a = b$$

Hence, *f* is a one-to-one.

Step 2: Solve the equation $y = f(x)$ for *x*.

$$y = 3x - 4$$
$$3x = y + 4$$
$$x = \frac{y+4}{3}$$

Step 3: Interchange *x* and *y*

$$y = \frac{x+4}{3} = f^{-1}(x)$$

Example 2: Show that the function $f(x) = \frac{2x-3}{3x+4}$ **is one-to-one. Find its inverse.**

Solution:

Step 1:

$$f(a) = f(b)$$
$$\Rightarrow \frac{2a-3}{3a+4} = \frac{2b-3}{3b+4}$$
$$\Rightarrow 6ab + 8a - 9b - 12 = 6ab + 8b - 9a - 12$$
$$\Rightarrow 8a - 9b = 8b - 9a$$
$$\Rightarrow 17a = 17b$$
$$\Rightarrow a = b$$

Hence, *f* is a one-to-one.

Step 2: Solve the equation $y = f(x)$ for *x*.

$$y = \frac{2x-3}{3x+4} \Rightarrow 3xy + 4y = 2x - 3$$
$$3xy - 2x = -4y - 3$$
$$x(3y - 2) = -4y - 3$$
$$x = \frac{-4y-3}{3y-2}$$

Step 3: Interchange *x* and *y*

$$y = \frac{-4x-3}{3x-2} = f^{-1}(x)$$

<u>Class Activity</u>

1) Find the inverse of the following functions:

i) $f = \{(a,1),(b,2),(c,3)\}$

Ans:

ii) $g = \{(1,5),(2,6),(3,7),(4,8)\}$

Ans:

2) Show that the function $f(x) = 2x + 3$ is one to one and find the inverse.

Solution:

3) Show that the function $f(x) = 4x - 3$ is one to one and find the inverse.

Solution:

4) Find the inverse of the following one to one function $f(x) = \frac{3x+4}{2}$

Solution:

5.4 OPERATIONS OF FUNCTIONS

Arithmetic of Functions

1). **Sum function**

$$(f+g)(x) = f(x) + g(x)$$

2). **Difference function**

$$(f-g)(x) = f(x) - g(x)$$

3) **Product function**

$$(f.g)(x) = f(x).g(x)$$

4) **Quotient function**

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$$

Example 1) Let f and g *be* the functions defined by $f(x) = x^2$-1 and $g(x) = x + 2$. Find the functions *i)* $f+g$ ii) $f-g$ iii) $f.g$ iv) $\frac{f}{g}$

Solution:

i) $(f+g)(x) = f(x) + g(x)$
$= (x^2 - 1) + (x + 2)$
$= x^2 + x + 1$

ii) $(f-g)(x) = f(x) - g(x)$
$= (x^2 - 1) - (x + 2)$
$= x^2 - 1 - x - 2$
$= x^2 - x - 3$

iii) $(f.g)(x) = f(x).g(x) = (x^2-1)(x + 2)$
$= x^3 + 2x^2 - x - 2$

iv) $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{x^2-1}{x+2}$

Example 2) Let 'f' and 'g' be the functions defined by $f(x) = \sqrt{2x+4}$ and $g(x) = \sqrt{x-2}$. Find the functions

i) $(f+g)(6)$
ii) $(f-g)(6)$
iii) $(f.g)(2)$
iv) $\left(\frac{f}{g}\right)(4)$

Solution:

i) $(f+g)(6) = f(6) + g(6)$
$= \sqrt{(12+4)} + \sqrt{(6-2)}$
$= \sqrt{16} + \sqrt{4}$
$= 4 + 2 = 6$

ii) $(f-g)(6) = f(6) - g(6)$
$= \sqrt{(12+4)} - \sqrt{(6-2)}$
$= 4 - 2 = 2$

iii) $(f.g)(2) = f(2).g(2)$
$= \sqrt{(4+4)}\sqrt{(2-2)}$
$= \sqrt{8}.0 = 0$

iv) $\left(\frac{f}{g}\right)(4) = \frac{f(4)}{g(4)} = \frac{\sqrt{8+4}}{\sqrt{4-2}}$
$= \frac{\sqrt{12}}{\sqrt{2}} = \sqrt{6}$

Example 3)*: If* $f(x) = ax + 25$ and $f(3) = 7$, find a.

Solution:

$f(x) = ax + 25$
$f(3) = a(3) + 25 = 7$
$3a + 25 = 7$
$3a = 7 - 25$
$3a = -18$
$a = \frac{-18}{3}$
$a = -6$

Class Activity

1) If f and *g be* the functions defined by

$f(x) = 2x + 3$, $g(x) = x^2 + 2$, then find

i) $(f + g)(x)$

ii) $(f - g)(2)$

iii) $(f.g)(2)$

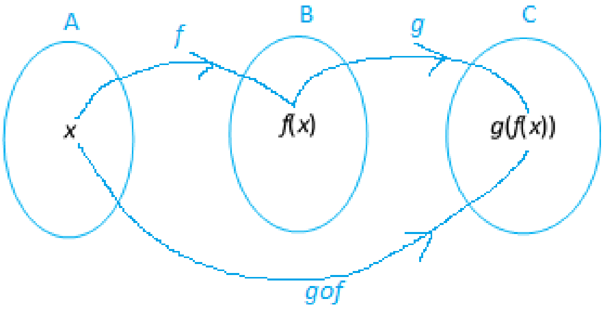
iv) $\left(\frac{f}{g}\right)(2)$

v) $2f(-1) + 3g(1)$

(2) If $f(x) = ax + 15$ any $f(5) = 10$.
Find 'a'
Solution:

5.5 COMPOSITE FUNCTION

Let f and g be functions , then $g \circ f$ is called the composite of g and f and is defined by the equation $(g \circ f)(x) = g(f(x))$.



Example 1:

Let 'f' and 'g' be the functions defined by $f(x) = 4x + 3$ and $g(x) = x^2$. Find $(fog)(x)$ and $(gof)(x)$.

Solution:

$$(gof)(x) = g(f(x)) = g(4x+3) = (4x+3)^2$$

and $$(fog)(x) = f(g(x)) = f(x^2) = 4x^2 + 3$$

Note: Here $(gof)(x) \neq (fog)(x)$

When we reverse the order of the function the result is not always the same. So be careful which function comes first.

Example 2:

Let 'f' and 'g' be the functions defined by $f(x) = \sqrt{9 - x}$ and $g(x) = 10 - x$

Find $(fog)(x)$ and $(fog)(4)$.

Solution: $$(fog)(x) = f(g(x)) = f(10 - x) = \sqrt{9 - (10 - x)} = \sqrt{x - 1}$$

$$(fog)(4) = \sqrt{4 - 1} = \sqrt{3}$$

Example 3:

Let 'f' be the functions defined by $f(x) = 5x - 2$. Find $(fof)(x)$ and $(fof)(3)$

Solution: $$(fof)(x) = f(f(x)) = f(5x - 2) = 5(5x - 2) - 2 = 25x - 10 - 2 = 25x - 12$$

$$(fof)(3) = 25(3) - 12 = 75 - 12 = 63$$

<u>Class Activity</u>

1) For the indicated functions 'f' and 'g', find the functions *fog*, *gof* and *fof*.

(i) $f(x) = 4 - x;\ \ g(x) = 2x^{\frac{1}{3}}$

Solution:

(ii) $f(x) = x^2 - 5x;\ g(x) = x^2 + 1$
Solution:

2) Let '*f*' and '*g*' be the functions defined by $f(x) = x + 1$ and $g(x) = x - 1$.

Find $g(f(-2))$

Solution:

3) Let '*f*' and '*g*' be the functions defined by $f(x) = x - 1$ and $g(x) = 2x + 5$.

Find $(f \circ g)(-1)$

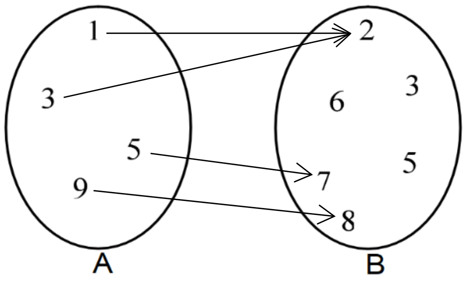
Solution:

WORKSHEET 5

Section-A

Circle the correct answer in the following questions.

(1) In the following figure the codomain is …..



(a) $\{1, 3, 5, 9\}$

(b) $\{2, 3, 5, 6, 7, 8\}$

(c) $\{2, 7, 8\}$

(2) The range in the set of ordered pairs $\{(1,1),(2,1),(3,2),(4,5)\}$ is …………..

(a) $\{1, 2, 5\}$

(b) $\{1, 2, 3, 4\}$

(c) $\{1, 2, 3, 4, 5\}$

(3) Which of the following ordered pairs are functions?

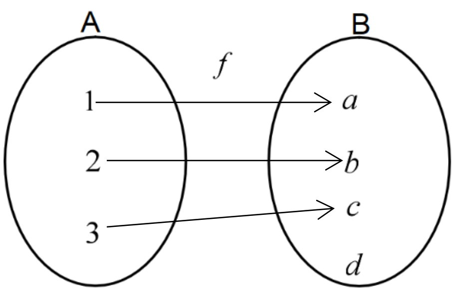
(a) $\{(b,1),(b,2),(b,3)\}$

(b) $\{(a,2),(b,2),(c,2)\}$

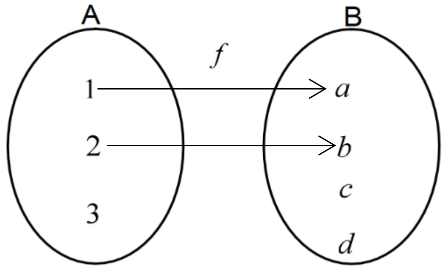
(c) $\{(1,1),(1,2),(2,3)\}$

(4) Which of the following is a function?

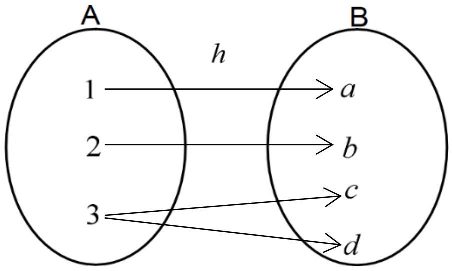
(a)



(b)

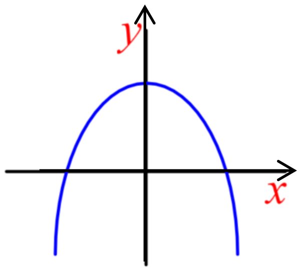


(c)

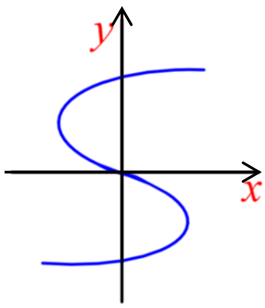


(5) Which of the following is a function?

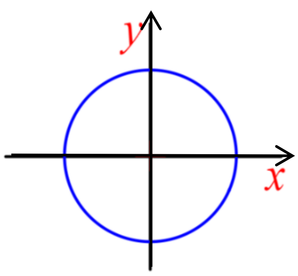
(a)



(b)



(c)



(6) Which of the following sets represent one to one function?

(a) $\{(1,a),(2,b),(3,b)\}$

(b) $\{(a,1),(b,1),(c,1)\}$

(c) $\{(a,1),(b,2),(c,3)\}$

(7) Which of the following equations represent one to one function?

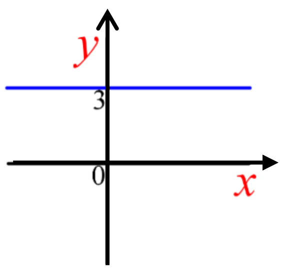
(a) $y = 3x + 2$

(b) $y = x^2 + 2$

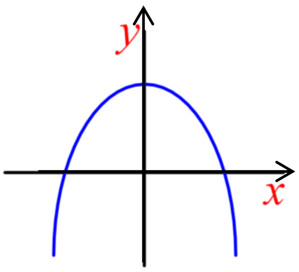
(c) $y = 2$

(8) Which of the following graphs shows a constant function?

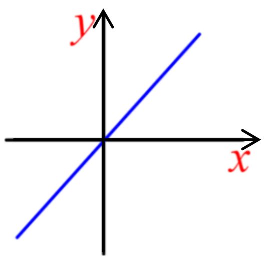
(a)



(b)



(c)



(9) Let ‘f’ be the function defined by $f(x) = x^2 + 1$, then $f\ (-2)$ is …………..

(a) 9

(b) 5

(c) 1

(10) If $f(x) = x - 4$, then $f^{-1}(x) =$

(a) 4

(b) $x - 4$

(c) $x + 4$

Section-B

Show your solution step by step in the following questions.

1) *If* $f(x) = x^2 - 3x,\ g(x) = 2x + 4$ then find

(i) $2f(-1) + 3g(-2)$

(ii) $\dfrac{f(-1)}{g(2)}$

(iii) $(f.g)(-1)$

(iv) $\left(\frac{f}{g}\right)(4)$.

2) Let 'f' and 'g' be the functions defined by $f(x) = 3x + 1$ and $g(x) = 2x - 3$ Find the functions

(i) $(f + \text{g})(x)$

(ii) $(f - g)(3)$

3) If $f(x) = 3x - 2$ find

(i) $\dfrac{f(x+h) - f(x)}{h}$

(ii) $\dfrac{f(x) - f(a)}{x - a}$

4) Show that the function $f(x) = \dfrac{x-5}{3}$ is one to one and find the inverse.

5) Find the inverse of the following one to one function $f(x)=\frac{x+2}{3x-1}$

6) Using $f(x) = x - 2$ and $g(x) = 5x + 3$ find

i) $f(g(x))$

ii) $g(f(-4))$

iii) $f(f(1))$

iv) $f(g(2))$

REFERENCES AND INDICATIVE READING

Title/Edition/Author	ISBN
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2. **College Algebra and Trigonometry-5th Edition** Margaret L. Lial, John Hornsby, David I. Schneider &Callie Daniels	ISBN-13: 978-0321671783 ISBN-10: 0321671783
3. **Bird's Basic Engineering Mathematics- 8th Edition** John Bird	ISBN-13: 978-0367643706 ISBN-10: 0367643707
4. **Engineering Mathematics- 8th Edition** K.A. Stroud and Dexter Booth	ISBN-13: 978-1352010275 ISBN-10: 1352010275
5. **Introduction to Statistics-3rd Edition** Ronald E. Walpole	ISBN-13 :**0024241405-978** ISBN-10 : **0024241407**

Websites:

i) http://www.statisticshowto.com

ii) http://math.tutorvista.com

iii) www.mathsisfun.com

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vi) https://www.hippocampus.org