Military Technological College

BASIC MATHEMATICS

WORKBOOK-2 **MODULE CODE: MTCG1016**

MILITARY TECHNOLOGICAL COLLEGE

Delivery Plan - Year 2023-24 [Term 1]

Title / Module Code / Programme	Basic Mathematics /MTCG1016/Foundation Programme Department (FPD)	Module Coordinator	Rajendar Palli
Lecturers	TBA	Resources & Reference books	Moodle & Workbook
Duration & Contact Hours	Term 1: 5 hrs x 11 weeks = 55 hours		

WEEK No.	TOPICS	HOURS	LEARNING OUTCOMES
1	**Introduction, Delivery of Material etc.**	1	1,2
	Basic Theory of Numbers & Operations 1.1 Basic Theory of Numbers 1.2 Arithmetic Operations and Fundamental Laws 1.2.1 The Four Basic Operations **Online Quiz 1** 1.2.2 Fundamental Laws of Operations 1.3 Directed Numbers and their Operations 1.4 Sequence of Arithmetic Operations **Online Quiz 2** **Worksheet 1-Moodle Online Test 1**	3	
	Set Theory 2.1 Sets, Types of Sets, Subsets, Venn Diagrams **Online Quiz 3**	1	
2	2.2 Union and intersection of Sets **Online Quiz 4** 2.3 Application of Set Theory Worksheet 2-Moodle Online Test 2	2	2, 3 and 8
	Basic Arithmetic 3.1 Factors and Multiples of a Number 3.1.1 Highest Common Factor (HCF) **Online Quiz 5** 3.1.2 Lowest Common Multiple (LCM) 3.1.3 Application of HCF & LCM **Online Quiz 6** 3.2 Reducing fractions to Simplest form 3.3 Addition and Subtraction of Fractions **Online Quiz 7**	3	

WEEK No.	TOPICS	HOURS	LEARNING OUTCOMES
3	3.4 Multiplication and Division of Fractions **Online Quiz 8** 3.5 Estimation/Rounding off 3.6 Scientific Notation **Online Quiz 9 & Worksheet 3- Moodle Online Test 3**	2	3, 6 and 8
	Basic algebra (Part-1) 4.1 Power Number Algebra and Laws of Indices **Online Quiz 10** 4.2 Algebra- Use of Symbols & Substitution 4.3. Addition and Subtraction of Polynomials 4.4 Multiplication of Polynomials **Online Quiz 11** **Worksheet 4 – Moodle Online Test 4**	3	
	Revision for CA1		
	Continuous Assessment 1(Topics: Units 1, 2 and 3)		1,2,3,6,8
4	**Techniques of Factorization and Rational Expressions** 5.1 Factorisation of Polynomials **Online Quiz 12** 5.2 Simplification of Rational Expressions 5.3 Multiplication and Division of Rational Expressions **Online Quiz 13** 5.4 Addition and Subtraction of Rational Fractions 5.5 Rationalising denominators of Irrational Expressions **Online Quiz 14** **Worksheet 5 – Moodle Online Test 5**	4	3, 4, 6, 8
	Units of Measurements, Percentages and Ratios 6.1 and 6.1.1 Units of Measurements and Conversions **Online Quiz 15**	1	
5	6.1.2 Inter-system conversions 6.1.3 Measuring Temperature **Online Quiz 16** 6.2 Percentages **Online Quiz 17** 6.3 Ratio and Proportion 6.3.1 Ratio and 6.3.2 Rate **Online Quiz 18** 6.3.3 and 6.3.4 Direct Proportion and Inverse Proportion 6.4 Map Scales (Online Quiz 19) **Worksheet 6-Moodle Online Test 6**	4	3,5,8
	Linear Equations, Inequalities and their Applications 7.1 Solving Linear Equations	1	

WEEK No.	TOPICS	HOURS	LEARNING OUTCOMES
6	7.2 Simultaneous Linear Equations 7.2.1 Solution by Elimination Method **Online Quiz 20** 7.2.2 Solution by Substitution Method 7.3 Linear Inequalities 7.3.1 Methods of describing inequalities **(Online Quiz 21)** 7.3.2 Solving linear inequalities **(Online Quiz 22)** **Worksheet 7 – Moodle Online Test 7**	3	5, 9
	Modelling Simple Real Life Problems 8.1 Word problems on linear equations 8.2 Word problems on linear Inequalities **Online Quiz 23** **Worksheet 8 – Moodle Online Test 8**	2	
7	**Quadratic Equations and Formulas** 9.1 Solving Quadratic equations 9.1.1 and 9.1.2 Solutions by Factorisation and Formula **Online Quiz 24** 9.2 Formation of Quadratic Equations 9.3 Equations involving radicals **(Online Quiz 25)** 9.4 Formula Transposition/subject change in Formula **Online Quiz 26** **Worksheet 9-Moodle Online Test 9**	3	7
	Revision of Units 4 to 8		
	Continuous Assessment 2(Topics: Units 4, 5, 6, 7 and 8)	2	3,4,5,6,8,9
8	**Angles and their measure** 10.1 Types of Angles 10.2 Conversion from radians to degrees and vice-versa **Online Quiz 27** 10.3 Length of an Arc and area of a Sector **Online Quiz 28** **Worksheet 10 – Moodle Online Test 10**	3	10,11,12,13
	Trigonometry 11.1.1 Definition and Properties of a Right Triangle. 11.1.2 The Pythagoras Theorem **Online Quiz 29**	2	

WEEK No.	TOPICS	HOURS	LEARNING OUTCOMES
9	11.2 Trigonometric ratios in a Right Triangle 11.2.1 The Six Trigonometric Ratios **Online Quiz 30** 11.2.2 Fundamental Trigonometric Identities 11.2.3 Applications of Trigonometric Identities **Online Quiz 31** 11.3 Solutions of Right Triangles and Applications 11.4 Angles of Elevation and Depression **Online Quiz 32** **Worksheet 11- Moodle Online Test 11**	5	11,12,13
10	**Plane Coordinate Geometry** 12.1 The Rectangular Coordinate System 12.2 Distance between two points 12.3 Gradient or Slope of a line **Online Quiz 33** 12.4 Equation of a straight line 12.5 Drawing graph of the straight line function based on its equation 12.6 Parallel and perpendicular lines **Online Quiz 34** **Worksheet 12 – Moodle Online Test 12**	5	14
11	**The Circle & Symmetry** 13.1 Centre and radius of the circle, tangent lines **Online Quiz 35** 13.2 Symmetry of Graphs **Worksheet 13 -Online Test Online Test 13** **Revision of Units 9 – 13**	5	14,15
12	**Final Exam(Topics: Units 9, 10, 11, 12, 13)**		
	TOTAL NUMBER OF TEACHING HOURS	55	

Indicative Reading		
Title/Edition/Author	**Publisher**	**ISBN**
College Algebra with Trigonometry, (9th Edition 2010), Raymond A. Barnett, Michael Ziegler and Karl Byleen, David Sobecki,	McGraw Hill	9780077350109
Basic Engineering Mathematics, (8th Edition 2021), Bird J.	Routledge	9780367643676
Engineering Mathematics, (8th Edition 2020), Stroud K.A and Booth D.J,	Red Globe Press	9781352010275

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Contents

4.1 Power Number Algebra and Laws of Indices

Few conventions for multiplying terms:

1) In algebra we usually leave out the multiplication sign ×
2) Any numbers must be written before the letters and all letters should be written in alphabetical order.
3) No need to write a 1 in front of the letter.
4) Write letters in alphabetical order.

For example: $4 \times a = 4a$

$b \times 5 = 5b$

$1 \times b = b$

$d \times 3 \times c = 3cd$

$6 \times e \times e = 6e^2$

Exponential/index notation:

$x \times x \times x \times x \times x = x^5$

read as '*x* to the power of 5'

This is called **exponential notation or index notation**. It is a short way of writing the same alphabet (or number) multiplied by itself many times.

For x^n we call *x* as base and *n* as exponent or index or power

Laws/rules of Indices

Rule 1: $a^0 = 1$

Any number, except 0, whose index is 0 is always equal to 1, regardless of the value of the base.

Example 1: $2^0 = 1$, $129^0 = 1$, $2678^0 = 1$, etc.

Rule 2: $a^{-m} = \frac{1}{a^m}$

Example 2: Simplify 2^{-2}

$$2^{-2} = \frac{1}{2^2} = \frac{1}{4}$$

Rule 3: $a^m \times a^n = a^{m+n}$

To multiply numbers with the same base, write the base and add the indices.

Example 3: Simplify 5×5^3

Solution: $5^1 \times 5^3 = 5^{1+3}$: (*note:* $5 = 5^1$)

$= 5^4$

Rule 4: $a^m \div a^n = a^{m-n}$

To divide numbers with the same base, copy the base and subtract the indices.

Example 4: Simplify $5(y^9 \div y^5)$

Solution: $5(y^9 \div y^5) = 5(y^{9-5})$

$= 5y^4$

Rule 5: $(a^m)^n = a^{mn}$

To raise a number to the n^{th} index, copy the base and multiply the indices.

Example 5: Simplify $(y^2)^6$

Solution: $(y^2)^6 = y^{2x6}$

$= y^{12}$

Rule 6: $a^{\frac{m}{n}} = \sqrt[n]{a^m}$ or $\left(\sqrt[n]{a}\right)^m$

Example 6: Simplify $125^{\frac{2}{3}}$

Solution: $125^{\frac{2}{3}} = \sqrt[3]{125^2}$

$= \left(\sqrt[3]{125}\right)^2$

$= 5^2 = 25$

Rule 7: $(ab)^n = a^n b^n$

Example 7: $(2 \times a)^2 = 2^2 \times a^2 = 4a^2$

Rule 8: $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

Example 8: $\left(\frac{3}{2}\right)^2 = \frac{3^2}{2^2} = \frac{9}{4}$

Class Activity 4.1

Simplify the following using laws of indices

1) $16\,a^0$

2) $\frac{b^{-2}}{b^2}$

3) $\frac{5}{x^{-3}}$

4) $\left(\frac{2}{3}\right)^{-3}$

5) $\frac{32p^2}{4p^8}$

6) $(2t^4)^3$

7) $(27)^{\frac{2}{3}}$

8) $\frac{3^n 9^{n-3}}{27^{n-1}}$

4.2. Algebra - Use of symbols & Substitution

Algebra is from Arabic word "al-jabr" meaning "reunion of broken parts".

Some examples of algebraic expressions:

$n + 7$: read as 'n plus 7'

$5 - n$: read as '5 minus n'

$2n$: read as '2 lots of n' or '2 times n'

$\frac{6}{n}$: read as '6 divided by a number n'

$4n + 5$: read as '4 times n plus 5'

x^3 : read as 'x with power or index 3'

$3(n + 4)$: read as'n plus 4 and then times 3'

Substitution: In algebra, when letters (alphabets) are replaced with numbers, it is called *substitution*.

Example 1: Evaluate the expression

$4 + 3n$ by substituting $n = 5$ and $n = 11$.

Solution:

when $n = 5$; $4 + 3n = 4 + 3\,(5)$

$= 4 + 15 = 19$

when $n = 11$; $4 + 3n = 4 + 3\,(11)$

$= 4 + 33 = 37$

Class Activity 4.2

Evaluate the following expressions:

when $a = 5, b = 2$ and $c = -1$

1) ab^2c^3

2) $a(b + c)$

3) $\frac{a+b-c}{a-b+c}$

4) $\frac{b^2-c^2}{a^2}$

4.3 Addition and subtraction of Polynomials

Polynomials: An algebraic expression involving only the operations of addition, subtraction, multiplication and raising to natural powers on variables and constants is called a polynomial. **A few examples are given below:**

i) $3x - 2$
ii) $x + 2y$
iii) 7
iv) $4x^2 - 2x + 5$
v) $x^3 - 3x^2y + 2xy^2 + 3y^4$

Note:
1) In a polynomial, a variable cannot appear in a denominator (like $\frac{1}{x}$), as an exponent (like 2^x), or within a radical (such as $(\sqrt{x}\,)$.

Examples of non-polynomials:

i) $\sqrt{2x} + \frac{2}{3x} - 4$

ii) $\frac{2x^2 - 3x + 2}{x + 3}$

iii) $\sqrt{2x^2 - 3x + 2}$

2) Single term polynomial is called a monomial, a two term polynomial is called a binomial and a three term polynomial a trinomial.

Examples:
i) $2xy^2$ is monomial
ii) $3x - 2$ is binomial
iii) $4x^2 - 2x + 5$ is trinomial

3) A constant in a term of a polynomial, including the sign that precedes it, is called the **coefficient** of the term. For example in the polynomial, $2x^4 - 4x^3 + x^2 - x + 5,$ the coefficient of the first term is 2, the coefficient of the second term is -4, the coefficient of the third term is 1, the coefficient of the fourth term is -1 and the last term is 5.

Degree of polynomial: Polynomial forms can be classified based on their degree. The degree of a polynomial is the highest power of the variable present in the expression.

If the polynomial contains two or more variables then the highest sum of the powers of the variables is called the degree of the expression.

Examples:

1) The degree of the first term in $2x^3 + \sqrt{3}x + \frac{2}{3}$ is 3, the degree of the second term is 1, the degree of the third term is 0 and **the degree of the whole polynomial is 3.**

2) The degree of the first term in $7x^2y^3 - \sqrt{2}xy^2z$ is 5, the degree of the second term is 4 and **the degree of the whole polynomial is 5**.

Note: i) Any non-zero constant is defined to be polynomial of degree 0.

ii) The number '0' is also a polynomial but is not assigned a degree.

Examples of polynomials and variables

1) Polynomials in one variable:

i) $x^2 + 2x + 1$

ii) $2x^3 + \sqrt{3}x + \frac{2}{3}$

2) Polynomial in several variables:

i) $3x^2 + 5xy - y^2$

ii) $7x^2y^3 - \sqrt{2}xy^2z$

Simplifying polynomials

Only like terms can be added or Subtracted and it is advisable to arrange the terms of the expression in descending powers.

Example:

$2x^4 - 5x^3 + 3x^4y - x^4y$

$= 2x^4 - 5x^3 + 2x^4y$

Class Activity 4.3

(1) In the polynomial $x^3 + 2x^2 + x - 1$
The coefficient of x^3 is

a) 1

b) 0

c) 3

(2) The degree of the polynomial

$x^2y^3 - x^2 + 2y^2$ is

a) 3

b) 5

c) 2

(3) Simplify:

$10x(x^3 - 3x + 1) + 2x^2(15 - 5x^2)$

(4) Simplify: $5x^2 + 3x - 1 - (2x^2 - 3x - 1)$

(5) Subtract

$3x(6x^2 - 4x - 2)$ *from* $2x^2(9x - 6)$

4.4 Multiplication of Polynomials

Multiplication or expansion of polynomial expressions.

By expanding brackets we mean **multiplying** each term in the bracket by the term outside.

Multiplication or expansion of polynomial expressions is distributive.

For example : $a(b+c) = a \times b + a \times c$

$$= ab + ac$$

and $(a+b)(c+d) = a(c+d) + b(c+d)$

$$= ac + ad + bc + bd$$

Remember
(+) (+) = + , (-) (-) = +
(+) (-) = - , (-) (+) = -
Example 1: Expand and simplify

$$3x(x+1) + 2(x-3)$$

Solution:

$$3x(x+1) + 2(x-3) = 3x^2 + 3x + 2x - 6$$

$$= 3x^2 + 5x - 6$$

Note: Simplification is based on the principle of grouping like terms together.

Example 2: Expand and simplify

$$(a-2)(a-3)$$

Solution:

$$(a-2)(a-3) = a^2 - 3a - 2a + 6$$

$$= a^2 - 5a + 6$$

The following rules will be used in the class activity questions and worksheet questions:

$(a+b)^2 = a^2 + 2ab + b^2$

$(a-b)^2 = a^2 - 2ab + b^2$

$(a+b)(a-b) = a^2 - b^2$

Class Activity 4.4

Expand and simplify the following:

(1) $(2x+3)^2$

(2) $(3x-5)^2$

(3) $(3x+2)^2 - (2x-3)^2$

(4) $(2x^2-3)\,(5x^2+3x-1)+(1-x^2)$

Worksheet 4

1) Simplify the following:

(i) $(2x^7y)(6x^4y^3)$

(ii) $81a^7b^3 \div 9a^2b$

(iii) $(x^2)^2 \div (y^3)^{\frac{2}{3}}$

(iv) $(27x^{-6}y^9)^{\frac{1}{3}}$

(v) $\frac{2^n 4^{n-1}}{8^n}$

2) If $a = 2, b = 1$ and $c = -1,$ then

find the value of $\sqrt{b^2 - 4ac}$

3) If $a = 3, b = -1$ and $c = 2$ then,

Find the value of $\frac{a^2-b^2}{a+b+c}$

4) Simplify the following:

i) $(6x^2 - x + 5) - 2x(3x - 2)$

ii) $2x(6x^2 + 2x - 1) - 3x(4x^2 - x + 1)$

5) What should be added to $a^3 + 3ab + 2b^2$ to obtain $5a^2 + ab$?

6) If $b = (2x + 2)$, $a = (x + 1)$ and $c = (x - 1)$, find $b^2 - 4ac$?

7) Simplify

$(x - y)(x - y)(x - y)$

Unit-5) Techniques of Factorisation and Rational Expressions

5.1 Factorisation of Polynomials

In algebra we often need to simplify expressions, and this may involve **expanding/removing** brackets as well as **factorising**.

Factorising is simply the reverse of expanding brackets.

5.1.1 Factorising by Inspection of HCF

This technique of factorisation is the reverse of the distributive multiplication:

$a(b+c) = ab + bc$.

This means that $ab + ac = a\,(b+c)$.

Example 1: Factorise $8x - 20$

Solution: $8x - 20 \;=\; 4(2x - 5)$,

Here **HCF = 4**

Example 2: Factorise $8x - 4y - 20$

Solution: $8x - 4y - 20 \;=\; 4(2x - y - 5)$,

Here **HCF = 4**

Example 3: Factorise $4x^2 - x$

Solution: $4x^2 - x = x(4x - 1)$

Here **HCF** is x.

Example 4: Factorise $2x^2 + 10x$

Solution: $2x^2 + 10x = 2x(x+5)$

Here **HCF** is $3x$.

<u>Note:</u>

There will not always be a highest common factor (HCF) or a common factor in all polynomial expressions given.

For example, the expression: $\boldsymbol{2x + 3y - 10}$ has no common factor.

5.1.2 Factorising a Difference of Two Squares

Recall that $(x+y)(x-y) = x^2 - y^2$, in which the left side are the factors, and the right side is the product that is written as a difference of two squares. Hence, to factorise a difference of two squares, we follow this rule:

$$\boxed{\boldsymbol{a^2 - b^2 = (a-b)(a+b)}}$$

Example 5: Factorise $x^2 - 16$

Solution: $x^2 - 16 = (x-4)(x+4)$

Example 6: Factorise $25x^2 - 9$

Solution: $25x^2 - 9 = (5x-3)(5x+3)$

5.1.3 Factorising by Grouping

Link: [Factorisation by grouping]()

Consider first two terms as one group and last two terms as another group and factorise separately.

Example 7: Factorise $2x + 4y - ax - 2ay$

Solution: $2(x+2y) - a(x+2y)$
$= (x+2y)(2-a)$

Example 8: Factorise $x^2 + 3x + 5x + 15$

Solution: $x(x+3) + 5(x+3)$
$= (x+3)(x+5)$

Example 9: Factorise $x^2 + 2x - 8x - 16$

Solution: $x(x+2) - 8(x+2)$
$= (x+2)(x-8)$

Example 10:
Suppose we have the expression
$x^2 + 7x + 10$

In this case we can **split the mid-term into two parts.**
$x^2 + 7x + 10 = x^2 + 2x + 5x + 10$

Now group the terms in pairs and factoring, we get: $x(x+2) + 5(x+2)$
$= (x+2)(x+5)$

Example 11: Factorise $2x^2 + 5x + 3$

Again, there is no common factor to all the three terms.

However, if we consider the product of the coefficient of the first term $2x^2$ and the constant 3 we get $2 \times 3 = 6$.

Now we need two factors of 6 which can be added to give 5, the coefficient of the mid- term.

Since 2 and 3 are factors of 6 and $2 + 3 = 5$, we can split the middle term $5x$ as $2x + 3x$. Hence, the expression

$$\begin{aligned} 2x^2 + 5x + 3 &= 2x^2 + 2x + 3x + 3 \\ &= (2x^2 + 2x) + (3x + 3) \\ &= 2x(x + 1) + 3(x + 1) \end{aligned}$$

Since the factor $(x + 1)$ is a common factor therefore:

$2x^2 + 5x + 3 = (x + 1)(2x + 3)$.

5.1.4 Factorising by Trial-and-Error Method

Another procedure to factorising a quadratic trinomial expression of the form $\boldsymbol{ax^2 + bx + c}$, where a, b and c have no common factors and $a = 1$ is as follows:

Step 1: write down two brackets with an x in each bracket and leave a space for the remaining terms:

$(x \quad\quad)(x \quad\quad)$

Step 2: Write down a set of factors for the product of a and c.

Step 3: Using all the possible factors for the product of a and c, expand all options to see which one gives you the correct middle term bx.

Note:

1. If c is positive, then the factors of c must be either both positive or both negative.
2. If c is negative, it means only one of the factors of c is negative, the other one being positive.
3. Once you get an answer, always multiply out your brackets again just to check or make sure it really works!

Example 12: Factorise $x^2 + 2x - 8$

Solution:

$x^2 + 2x - 8 = (x \quad\quad)(x \quad\quad)$

Now we check for factors the product of 1 and -8, which is -8 which can give us $+2$

The factors of -8 in pairs are

$(-1,8); (-8,1); (-2,4); (-4,2)$

But the pair which gives us $+2$ is $(-2,4)$ that is $-2 + 4$, so we can write in the blank spaces

$$\begin{aligned} x^2 + 2x - 8 &= (x \quad\quad)(x \quad\quad) \\ &= (x - 2)(x + 4) \end{aligned}$$

Therefore: $x^2 + 2x - 8 = (x - 2)(x + 4)$

Example 13: Factorise $3x^2 - 7x - 6$.

Solution:

Factorise the 1st term and the last term.

Factors of the 1st term: $3x^2$ *are* $3x$ *and* x

Factors of -6 are ±1, ±6, ±2, and ±3.

Trial 1: $3x^2 - 7x - 6 = (3x \quad\quad)(x \quad\quad)$

$= (3x + 2)(x - 3)$

Check by multiplication using F- O -I - L Method.

Note: The middle term is the sum of the products of outside terms and the product of the inside terms.

$$\begin{aligned} (3x + 2)(x - 3) &= 3x^2 + (-9x + 2x) - 6 \\ &= 3x^2 - 7x - 6 \end{aligned}$$

Therefore, $3x^2 - 7x - 6 = (3x + 2)(x - 3)$.

[**Link: Factorising Quadratic Expressions**]

Class Activity 5.1

1) Factorise the following:

i) $6x + 12$

ii) $8a + 12b - 4c$

iii) $3x^2 + 6x$

2) Factorise: $9a^2 - 25$

3) Factorise the following by grouping:

i) $3x^2 + 6x + 5x + 10$

ii) $2x^2 + 10x - 5x - 25$

4) Factorise the following:

i) $x^2 + 12x + 20$

ii) $x^2 - 4x - 21$

5) Factorise: $2x^2 + 9x - 18$

5.2 Simplification of Rational Expressions

Definition: Rational expression:

One algebraic expression divided by another algebraic expression is called a **Rational** expression.

Examples: $\frac{5x}{4y}$, $\frac{2x-3y}{3x+4y}$, $\frac{2x+3}{x^2-5x+8}$

Simplification of Rational Expressions

We can simplify or reduce a fraction by dividing out the common factors that occur in both the numerator and denominator.

Example (1): Simplify $\frac{12x^3y^4}{18x^2y^5}$

Solution:

$$\frac{12x^3y^4}{18x^2y^5} = \frac{\cancel{6} \times 2 \times \cancel{x} \times \cancel{x} \times x \times \cancel{y} \times \cancel{y} \times \cancel{y} \times \cancel{y}}{\cancel{6} \times 3 \times \cancel{x} \times \cancel{x} \times y \times \cancel{y} \times \cancel{y} \times \cancel{y} \times \cancel{y}} = \frac{2x}{3y}$$

Canceling common terms.

Example (2): Simplify $\frac{5x+10}{3x^2+6x}$

Solution: $$\frac{5x+10}{3x^2+6x} = \frac{5\cancel{(x+2)}}{3x\cancel{(x+2)}} = \frac{5}{3x}$$

Example (3): Simplify $\frac{2a+8}{a^2+6a+8}$

Solution: $$\frac{2a+8}{a^2+6a+8} = \frac{2\cancel{(a+4)}}{(a+2)\cancel{(a+4)}} = \frac{2}{a+2}$$

Example (4): Simplify $\frac{3x-6}{x^2-4}$

Solution: $$\frac{3x-6}{x^2-4} = \frac{3\cancel{(x-2)}}{(x+2)\cancel{(x-2)}} = \frac{3}{x+2}$$

Example (5): Simplify $\frac{x^2-7x+10}{x^2+x-6}$

Solution: $$\frac{x^2-7x+10}{x^2+x-6} = \frac{\cancel{(x-2)}(x-5)}{\cancel{(x-2)}(x+3)} = \frac{x-5}{x+3}$$

Class Activity 5.2

1) Simplify: $\frac{15x^2y^3}{20x^3y^5}$

2) Simplify: $\frac{4x^2+12x}{5x^2+15x}$

3) Simplify: $\frac{3a-9}{a^2-8a+15}$

4) Simplify: $\frac{3x^2-27}{2x+6}$

5) Simplify: $\frac{x^2-4x+4}{x^2+5x-14}$

5.3 Multiplication and Division of Rational Expressions

Just as we can multiply and divide fractions, we can also multiply and divide rational expressions that include polynomials.

5.3.1 Multiplication of Rational Expressions

1. Factorise all numerators and denominators.
2. Cancel all common factors.
3. Either we multiply all the numerators together and all the denominators together or leave the answer in factored form.

Example (1): $\left(\frac{6a^2}{5b^2}\right)\left(\frac{10b^3}{3a^4}\right)$

Solution: $\left(\frac{6a^2}{5b^2}\right)\left(\frac{10b^3}{3a^4}\right) =$

$$\left(\frac{2\times\cancel{3}\times\cancel{a}\times\cancel{a}}{\cancel{5}\times\cancel{b}\times\cancel{b}}\right)\left(\frac{2\times\cancel{5}\times\cancel{b}\times\cancel{b}\times b}{\cancel{3}\times\cancel{a}\times\cancel{a}\times a\times a}\right) = \frac{4b}{a^2}$$

Alternatively:

$$\left(\frac{6a^2}{5b^2}\right)\left(\frac{10b^3}{3a^4}\right) = \frac{6\times 10}{5\times 3}\ a^{2-4}b^{3-2}$$

$$= 4a^{-2}b$$

Example (2): $\left(\frac{3x-6}{x+3}\right)\left(\frac{5x+15}{x-2}\right)$

Solution: $\left[\frac{3\cancel{(x-2)}}{\cancel{x+3}}\right]\left[\frac{5\cancel{(x+3)}}{\cancel{x-2}}\right] = 15$

Example (3): $\left(\frac{x^2+6x+9}{x^2-9}\right)\left(\frac{3x-9}{x^2+2x-3}\right)$

Solution: $\left[\frac{(x+3)^2}{(x+3)(x-3)}\right]\left[\frac{3(x-3)}{(x+3)(x-1)}\right]$

$$= \left[\frac{\cancel{(x+3)}\cancel{(x+3)}}{\cancel{(x+3)}\cancel{(x-3)}}\right]\left[\frac{3\cancel{(x-3)}}{\cancel{(x+3)}(x-1)}\right]$$

$$= \frac{3}{x-1}$$

5.3.2 Division of Rational Expressions

1. When we divide rational expressions, we multiply the first fraction (dividend) by the reciprocal of the second fraction (divisor).
2. Factorise all numerators and denominators.
3. Cancel all common factors.
4. Either multiply the denominators and numerators together or leave the solution in factored form.

Example (4): $\frac{5x^2}{9} \div \frac{15x^3}{18}$

Solution: $\frac{5x^2}{9}\times\frac{18}{15x^3}$

$$= \frac{\cancel{5}\times\cancel{x}\times\cancel{x}}{\cancel{3}\times\cancel{3}}\times\frac{2\times\cancel{3}\times\cancel{3}}{3\times\cancel{5}\times\cancel{x}\times\cancel{x}\times x} = \frac{2}{3x}$$

Alternatively:

$$\frac{5x^2}{9}\times\frac{18}{15x^3} = \left(\frac{5\times 18}{9\times 15}\ x^{2-3}\right)$$

$$= \frac{2}{3}\ x^{-1}$$

Example (5): $\left(\frac{5x^2}{x+3}\right)\div\left(\frac{10x^4}{x^2+5x+6}\right)$

Solution: $\left(\frac{5x^2}{x+3}\right)\times\left(\frac{x^2+5x+6}{10x^4}\right)$

$$= \left(\frac{\cancel{5}\times\cancel{x}\times\cancel{x}}{\cancel{x+3}}\right)\times\left(\frac{(x+2)\cancel{(x+3)}}{2\times\cancel{5}\times\cancel{x}\times\cancel{x}\times x\times x}\right)$$

$$= \frac{x+2}{2x^2}$$

Alternatively: $\left(\frac{5x^2}{x+3}\right)\times\left(\frac{x^2+5x+6}{10x^4}\right)$

$$= \left(\frac{5}{10}\right)\left(\frac{(x+2)(x+3)}{x+3}\right)x^{2-4} = \frac{1}{2}(x+2)\ x^{-2}$$

$$= \frac{1}{2}(x+2)\ x^{-2}$$

Example (6): $\left(\frac{x^2-9}{x^2-4}\right)\div\left(\frac{x^2-2x-15}{x^2-3x-10}\right)$

Solution: $\left(\frac{x^2-9}{x^2-4}\right)\times\left(\frac{x^2-3x-10}{x^2-2x-15}\right)$

$$= \left(\frac{\cancel{(x+3)}(x-3)}{\cancel{(x+2)}(x-2)}\right)\times\left(\frac{\cancel{(x+2)}\cancel{(x-5)}}{\cancel{(x+3)}\cancel{(x-5)}}\right)$$

$$= \frac{x-3}{x-2}$$

Class Activity 5.3

1) Simplify: $\left(\frac{10x^2}{3xy^2}\right)\left(\frac{9y^3}{15x}\right)$

2) Simplify: $\left(\frac{x^2-4}{x^2+2x-8}\right)\left(\frac{x^2+10x+24}{x^2-8x-20}\right)$

3) Simplify: $\left(\frac{3ab}{5x^2y^2}\right) \div \left(\frac{9a^2b^2}{10xy}\right)$

4) Simplify$\left(\frac{2x-4}{x^2-3x+2}\right) \div \left(\frac{1}{x-1}\right)$:

5.4 Addition and Subtraction of Rational Fractions

Recall: $\frac{a}{b} \rightarrow \frac{Numerator}{Denominator}$

If in a fraction numerator and/or denominator contains polynomial it is called **a rational expression.**

Common Steps to simplifying addition or subtraction of rational polynomial expressions.

Step 1: Find the Lowest Common Denominator (LCD).

Step 2: Rewrite the fractions as similar fractions based on the common denominator.

Step 3: Combine the fractions and simplify.

Example 1: Simplify: $\frac{2}{x-1} + \frac{4}{x+2}$

Solution: LCD= $(x-1)(x+2)$

$$\therefore \frac{2}{x-1} + \frac{4}{x+2} = \frac{2\,(x+2)}{(x-1)(x+2)} + \frac{4\,(x-1)}{(x-1)(x+2)}$$

$$= \frac{2(x+2)+4(x-1)}{(x-1)(x+2)}$$

$$= \frac{2x+4+4x-4}{(x-1)(x+2)}$$

$$= \frac{6x}{(x-1)(x+2)}$$

Example 2: Simplify $\frac{2}{x-1} + \frac{4x}{(x-1)^2}$

Solution: LCD = $(x-1)^2$ *or* $(x-1)(x-1)$

$$\frac{2}{x-1} + \frac{4x}{(x-1)^2} = \frac{2(x-1)}{(x-1)(x-1)} + \frac{4x}{(x-1)^2}$$

$$= \frac{2(x-1)+4x}{(x-1)^2}$$

$$= \frac{6x-2}{(x-1)^2}$$

Class Activity 5.4

Simplify the following:

1) $\frac{x^2}{x-1} - \frac{1}{x-1}$

2) $\frac{3}{x+3} - \frac{2x+3}{(x+2)(x+3)}$

3) $\frac{x}{x^2+2x+1} - \frac{1}{x+1}$

4) $\frac{x+3}{x^2+6x+9} - \frac{x-3}{x^2-6x+9}$

5) $\frac{x+2}{x^2+7x+10} + \frac{x-5}{x^2-25}$

Link: Adding & subtraction of rational expressions

5.5 Rationalising of Irrational Expressions

From laws of indices, $\sqrt[n]{b} = (b)^{\frac{1}{n}}$ where 'n' is a natural number and b a real number, is defined as the principal n^{th} root of b.

Note: The symbol $\sqrt{\ }$ is called a radical, n is called the index and b is called the radicand.

Surds: A **surd** is an expression that includes a square root, cube root or other root symbol.

Surds are used to write **irrational numbers** precisely - because the decimals of **irrational numbers** do not terminate or recur, they cannot be written exactly in decimal **form**.

Example: Numbers like $\sqrt{12}$ can be written as

$\sqrt{4 \times 3} = 2\sqrt{3}$ in Surd Form.

Properties of the radicals:

1) $\sqrt[n]{x^n} = x$

For example: $\sqrt[4]{16} = \sqrt[4]{2^4} = 2$

2) $\sqrt[n]{xy} = \sqrt[n]{x}\sqrt[n]{y}$

For example: $\sqrt[3]{6} = \sqrt[3]{2 \times 3} = \sqrt[3]{2}\sqrt[3]{3}$

3) $\sqrt[n]{\frac{x}{y}} = \frac{\sqrt[n]{x}}{\sqrt[n]{y}}$

For example: $\sqrt[3]{\frac{8}{27}} = \frac{\sqrt[3]{8}}{\sqrt[3]{27}} = \frac{2}{3}$

4) $\sqrt[n]{b^m} = b^{\frac{m}{n}}$

For example: $\sqrt[3]{15^2} = 15^{\frac{2}{3}}$

Rationalizing Fractions with irrational denominators:

Rationalising fractions with irrational denominators simply means finding a way of making the denominators without radicals.

This can be achieved by multiplying both the numerator and denominator by an irrational number or expression which can make it possible to simplify to a whole number.

Example: 1) Rationalise the fraction $\frac{3}{\sqrt{5}}$

Solution: $\frac{3}{\sqrt{5}} = \frac{3 \times \sqrt{5}}{\sqrt{5} \times \sqrt{5}} = \frac{3\sqrt{5}}{5}$

Example: 2) Rationalise $\frac{\sqrt{x}+\sqrt{y}}{\sqrt{x}-\sqrt{y}}$

Solution: $\frac{\sqrt{x}+\sqrt{y}}{\sqrt{x}-\sqrt{y}} = \frac{(\sqrt{x}+\sqrt{y})}{(\sqrt{x}-\sqrt{y})} \times \frac{(\sqrt{x}+\sqrt{y})}{(\sqrt{x}+\sqrt{y})}$

$$= \frac{(\sqrt{x}+\sqrt{y})^2}{x-y}$$

$$= \frac{x+2\sqrt{xy}+y}{x-y}$$

Class Activity 5.5

1. Write in surd form:

   i) $\sqrt{8}$

   ii) $\sqrt{27}$

2) In the following fractions, rationalize denominators and simplify leaving answers in surd form.

   i) $\frac{6}{\sqrt{2}}$

   ii) $\frac{3}{2-\sqrt{y}}$

   iii) $\frac{9-y}{3+\sqrt{y}}$

Worksheet-5

(1) Factorise: $5a - 10b + 20c$

(2) Factorise: $50x^2 - 8y^2$

(3) Factorise completely: $5 - 5a^2 + a^3 - a^5$

(4) Factorise: $x^2 - 3x - 40$

(5) Factorise: $x^2 + 5x - 36$

(6) Factorise: $6x^2 - 19x + 15$

(7) Factorise: $3x^2 - 5x - 12$

(8) Simplify: $\frac{12a^4b^3}{15a^6b}$

(9) Simplify: $\frac{3x^2-4x+1}{x^2-x}$

(10) Simplify: $\left(\frac{x^2-9}{x^2+3x}\right)\left(\frac{x}{2x-6}\right)$

(11) Simplify: $\left(\frac{4x+8}{3x-6}\right)\left(\frac{5x^2-10x}{10x+20}\right)$

(12) Simplify: $\left(\frac{4x^3y^3}{3a^2b^2}\right) \div \left(\frac{xy^2}{6ab^2}\right)$

(13) Simplify: $\left(\frac{2x^2-8}{x^2+4x-12}\right) \div \left(\frac{x-2}{x^2+6x}\right)$

(14) Simplify: $\frac{3}{x-3} - \frac{2}{3-x}$

(15) Simplify: $\frac{x+4}{x^2+6x+9} - \frac{1}{x+3}$

(16) Simplify: $\frac{2x-4}{x^2-4} - \frac{x+2}{x^2-2x-8}$

(17) Simplify: $\frac{4x}{x^2-y^2} + \frac{3}{x+y} - \frac{2}{x-y}$

(18) Write in Surd form:

$\sqrt{20}$

(19) Rationalize denominator and write in simplified form:

i) $\frac{10}{\sqrt{5}}$

ii) $\sqrt{27} - \frac{3}{\sqrt{3}}$

iii) $\frac{19}{3\sqrt{3}-2\sqrt{2}}$

(Unit 6) Units and Measurements

6.1 Conversions from one unit to another

Unit of measurement is a definite magnitude of a physical quantity.
Examples: kilometres (km), metres (m).

System International (SI) Units

The SI base units and their physical quantities are:

Base Quantity	Base Unit	Symbol
Length	Metre	m
Mass	Kilogram	kg
Time	Second	s
Electric Current	Ampere	A
Temperature	Kelvin	K
Amount of substance	Mole	mol
Intensity of light	Candela	cd

The examples of some of the **derived units** form the SI units are:

Derived Quantity	Derived Unit	Symbol
Area	square metre	m^2
Volume	cubic metre	m^3
Velocity	metre per second	$\frac{m}{s}$
Acceleration	metre per square second	$\frac{m}{s^2}$
Frequency	Hertz	$Hz = \frac{1}{s}$
Density	Kilogram per cube metre	$\frac{kg}{m^3}$
Momentum	Kilogram metre per second	$\frac{kg\,m}{s}$
Force	Newton	$1N = 1\frac{kg\,m}{s^2}$
Energy,	Joule	$J = N\,M$
Power	Watt	$W = \frac{J}{s}$
Pressure	Newton per square metre or Pascal	$Pa = \frac{N}{s^2}$

Note: Many other units, such as gram and litre, are formally not part of the SI, but are accepted for use with SI.

SI Prefixes

Prefix	Symbol	Value
Tera	T	10^{12}
Giga	G	10^{9}
Mega	M	10^{6}
Kilo	k	10^{3}
Hecto	h	10^{2}
Deca	da	10
Base unit		1
Deci	d	10^{-1}
Centi	c	10^{-2}
milli	m	10^{-3}
micro	μ	10^{-6}
nano	n	10^{-9}
Pico	P	10^{-12}

Note: Prefixes now range from yotta at 10^{24} to yocto at 10^{-24}.

6.1.1 Conversion of units of length and distance

It is often necessary to convert one metric unit to another.

Example 1: Convert 2.5 kilometres into the equivalent length in metres.

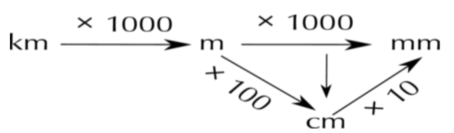
Solution: The factor needed to convert kilometres to metres can be developed like this:
We know that: $1km = 10^3 m$ (since kilo = 10^3)
or $1km = 1000m$
Multiplying both sides by 2.5 we get,
$$2.5 \times 1\ km = 2.5 \times 1000m$$
or $$2.5\ km = 2500\ m$$

Example 2) Convert 32 **centimetres** into the equivalent length in **kilometres**.

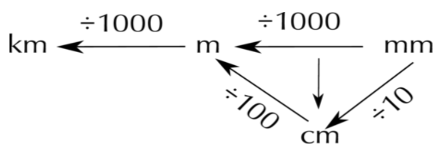
Solution:
Step (1) $32\ cm = \frac{32}{100} m = 0.32\ m$

Step (2) $0.32\ m = \frac{0.32}{1000} km = 0.00032\ km$

A quick way of looking at conversion of units of length:
1) **To convert larger units into smaller units**:



2) **To convert smaller units into larger units**:



Class Activity 6.1.1

Convert the following units of length.
Remember to show all of your calculations.

1) A leaf is 25 mm long. How long is it in cm?

2) A bolt is 3.2 cm long. What is its length in mm?

3) A sofa is 187 cm long. How long is it in metres?

4) Your College tennis court is 23.78 m long. How long is it in cm?

5) The distance between Sophie's house and the shop is $6359m$. Convert this distance into km.

6) Reggie and Lebo live 7.02 km apart. What is this distance in metres?

6.1.2 Inter-system conversions

The most often used **units of length** are:

- 1 inch = 2.54 cm (exactly)
- 1 metre = 39.37 inch (exactly)
- 1 foot = 12 inches
- 1 mile = 1.609 km (exactly)

The most often used **units of mass** are:

- 1 ton = 1000 kilogram (kg)
- 1 kg = 2.20 pounds (lbs) (3 Sig. Fig.)
- 1 lb = 454 g (3 Sig. Fig.)
- 1 ounce (oz) = 0.0625 lb

The most often used **units of volume** are:

- 1 milliliter (ml) =1cm^3 =1cc (cubic centimeter)
- 1 Litre (l or L) = 1.06 quart (qt) (3 Sig. Fig.)
- 1 qt = 946 mL (3 Sig. Fig.)
- 1 Imperial Gallon = 4.55 L (3 Sig. Fig.)

Note:

1) Above are merely commonly used units and not necessarily accepted for use in SI.
2) Sig. Fig. means **significant figures**.

Examples on conversions of units from one form to another:

1) Convert 1 inch to millimeters.

Solution:

$\mathbf{1\ inch = 2.54\ cm}$ and $\mathbf{1\ cm = 10\ mm}$

So 1 $inch = 2.54 \times 10mm = 25.4\, mm.$

For any number of inches, just multiply by 25.4 then put the unit mm.

2) Convert 10 pounds into kilograms.

Solution: Use $\mathbf{2.2\ lb = 1kg}$

Divide both sides by 2.2 and we get

$\mathbf{1lb = 0.454\ kg}$.

For any number of pounds, just multiply by 0.454 and put the unit kg.

10 pounds = $(10)(0.454) = 4.54\ kg$

3) Convert 20 gallons to litres.

Solution: We know,

1 Imperial gallon = 4.546 litres,

So, 20 Imperial gallons = 20×4.546

$= 90.92\ litres.$

Class Activity 6.1.2

Circle the correct answer for 1 - 5

1. To convert Imperial gallons to litres, multiply by

A) 4.55
B) 0.00455
C) 0.568.

2. 6 mm is equal to

A) 0.625 inches.
B) 0.236 inches.
C) 0.375 inches.

3. Express 750 milligrams in grams.

A) 0.75 grams
B) 7.5 grams
C) 75 grams

4. How many centimetres is in an inch?

A) 25.4
B) 2.54
C) 0.254

5. 5 miles is how many kilometres?

A) 1.6
B) 8
C) 10

6. What is 75 m/s in km/h ?

7. Convert 100 km/h to m/s

6.1.3 Measuring Temperature

There are three commonly used units for temperature, these are: Celsius, Kelvin and Fahrenheit.
They are merely different representations of the same temperature and can be converted into one another.

Below are the formulas to use:

1. $\boldsymbol{K = °C + 273.15}$
Used to change from degrees $Celcius(°C) to\ Kelvin(K)$ and vice versa.

2. $\boldsymbol{°F = (°C * 9/5) + 32 = 1.8\,°C + 32}$
Used to change from degrees $Celcius$ (°C) to degrees Fahrenheit (°F) and vice versa.

For example

Freezing point of water

$\boldsymbol{= 0°C = 273.15K = 32°F}$

Examples :
Make the following temperature conversions.
1) 135 °F = _______ °C

Explanation: From the relation

$$°C = (F - 32) * 5/9$$

$$or\ °C = (F - 32)/1.8$$

$$°C = (135 - 32) / 1.8 = 57.22$$

Answer: $\boldsymbol{57.2\,°C}$

2) 25 $°C$ = _____ K

Explanation: From the relation

$$K = C + 273.15$$

$$K = 25 + 273.15 = 298.15$$

Answer: $\boldsymbol{298.15\,K}$

3) 45 $°F$ = _____ K

Explanation: From the relation

$$K = (F - 32) * 5/9 + 273.15$$

$$K = (45 - 32) * 5/9 + 273.15$$

$$= 7.222 ... + 273.15 = 280.37$$

Answer: $\boldsymbol{280.37\,K}$ (up to 2 decimal places)

Class Activity 6.1.3

Make the following temperature conversions:

1) 45 $°C$ = _____ $°F$

2) 250 K = ______ $°C$

3) 140 $°F$ = _____ K

4) 250K = _______$°F$

6.2 Percentages

Percentage (%): *A **percentage** is a number or ratio that refers to any proportion or share in relation to a whole.*

When any fraction is expressed or written with a denominator of one hundred (100), it is called a percentage written with symbol "%",

- The % says, "per hundred."
- For example, 47%, is simply another way of writing the fraction $\frac{47}{100}$

So to write a percentage as a fraction, simply put it over 100 like this: $24\% = \frac{24}{100}$

Note: There is need to write the fraction in its simplest terms.

$$24\% = \frac{24}{100} = \frac{12}{50} = \frac{6}{25}$$

To change a percentage into a decimal divide by 100.

Example: $35\% = \frac{35}{100} = 0.35$

To change a decimal or a fraction into a percentage

To change a decimal or fraction into a percentage, multiply by 100 then affix "%".

Example: Convert 0.23 into percentage.

Solution: $0.23 \times 100 = 23\%$

Class Activity 6.2.1

1. **Complete this chart**

Question	Percentage	Fraction
1.	23%	$\frac{23}{100}$
2.	24%	
3.	55%	
4.	20%	
5.	17%	
6.	78%	

2. **Complete this chart**

Percentage	Decimal	Decimal	Percentage
1) 27%		7) 0.76	
2) 2%		8) 0.06	
3) 55%		9) 0.42	
4) 5%		10) 0.04	
5) 17%		11) 0.83	
6) 98%		12) 0.07	

Finding one quantity as a percentage of another quantity

Example 1: Jill obtained a score of 23 out of 32 in her marine engineering exam. She wants to know what this will be as a percentage.

Solution: To do this, you

- Write the result as a fraction $\frac{23}{32}$
- Change the fraction into a decimal $(23 \div 32 = 0.71875)$
- Change the decimal into a %
  $(0.71875 \times 100 = 71.9\%)$
  $\approx 72\%$

Example 2: What percentage of cars is green in the following table?

Car Park Survey	
Colour	**Frequency**
Green	3
Silver	5
Black	4

Solution:

Total is 12 and there are 3 green cars.

We want to express $\frac{3}{12}$ as a percentage.

$\frac{3}{12} \times 100\% = 25\%$

Example 3: In a survey of 120 pupils, it was found that 20% had personal stereos. How many pupils had personal stereos?

Solution: $\frac{20}{100} \times 120 = 24$ pupils.

Link: Finding percentage of a number

Percentage problems

Determining a percentage (%) of a given quantity.

General Method

Step 1) Change the percentage value into a decimal or express it as a fraction of 100.

Step 2) Change the word 'of' into a multiplication " × " sign.

Step 3) Multiply the fraction or decimal by the quantity involved.

Example 4: What is 23% of 45?

Solution: $0.23 \times 45 = \mathbf{10.35}$

or $\frac{23}{100} \times 45 = \mathbf{10.35}$

Example 5:
Express $200m$ as a percentage of $5km$.

Solution: Changing both values to the same unit gives $5km = 5000m$,

Expressing $200m$ as a fraction of $5000m$ and as a percentage we get $\frac{200}{5000} \times 100\% = 4\%$

Example 6: If an amount of $24 is increased by 25%, find new amount.

Solution: First method is to get 25% of $24 then add to the original value $24.

Since $\frac{25}{100} \times \$24 = \$6$

Thus, then the new amount becomes
$\$24 + \$6 = \$\mathbf{30}$

The second method is, we consider that the original amount of $24 is equivalent to 100%. After an increment of 25% the new amount is equivalent to 125% of the original amount. Therefore, to get the new amount we calculate:

$$\frac{125}{100} \times \$24 = \$\mathbf{30}$$

Example 7: Find a number so that 65% of it is 52.

Solution: 65% of x = 52

$$\frac{65}{100} \times x = 52$$

$$x = \frac{100}{65} \times 52 = 80$$

∴ The number is **80**

Class Activity 6.2.2

1) A student obtains 6 marks out of 15 in a quiz. What is his mark as a percentage?

2) 15% of 300 pens are defective. Find the number of defective and good pens.

3) An engineer turns up to work 135 times out of a possible 160 times. What % is his attendance?

4) After a 20% discount a mobile phone costs RO 50. What was the cost of the mobile phone before the discount?

6.3 Ratio and Proportion

6.3.1 Ratio: A ratio is the quantitative relation between two amounts showing the number of times one value contains or is contained within the other.
Example: The ratio of computers to students is 2 to 1 and it is written as $2:1$.

In a ratio there is a comparison of two or more numbers that are usually of the same type or measurement.

If the numbers have different units, it is important to convert the units to be the same before doing any calculations.

We write the numbers in a ratio with a colon (:) between them.

For **example**, if there are 8 students who are studying Marine Engineering and 12 students who are studying Civil Engineering, then we say we have a ratio of 8 students doing Marine Engineering to 12 students doing Civil Engineering.

We can write this as 8 : 12. We can also simplify this ratio to 2 : 3, by dividing both parts by 4.

Note: It is important in which order you state the ratio. A ratio of 1 : 7 is not the same as a ratio of 7 : 1.

For instance, $a:b = \frac{a}{b}$, hence it's not the same as $b:a = \frac{b}{a}$

Finding missing values in ratios

Ratios are commonly used in situations where some quantities need to be subdivided into parts according to some pre-determined way.

For example, suppose to make brass (an alloy of copper and zinc) a chemical engineer wants to have 65% copper and 35% zinc. For a final alloy with a mass of $40g$, what is the mass of copper and zinc needed?

Solution: Method 1:

65% of $40g$ means $\frac{65}{100} \times 40 = 26g$ of copper and the remaining 35% is equal to $(40 - 26)g = 14g$ of zinc.

Link:Ratios

Method 2:
Consider the ratio of the percentages as $65:35 = 13:7$ when simplified by 5
The total ratio is $13 + 7 = 20$
Hence to get the amount of copper needed we have: $\frac{13}{20} \times 40g = 26g$, and the other $\frac{7}{20} \times 40g = 14g$ will be for zinc.

Class Activity 6.3.1

1) Split the following quantities into the given ratios:
   i) 48 in to the ratio 3:5
   ii) 240 into the ratio 5:7

2) Salim and Ahmed invest RO 3400 in their bank. They put the money in the ratio 3:7.
   i) How much did each person put in?
   ii) After a year, the money accrues interest. They withdraw RO 6200 and split the new amount in the same ratio. How much does each get?

3) A compound is formed by combining iron, chromium and carbon in the ratio $1:2:3$ If 20g of chromium is used, what is the mass of iron and carbon that should be used?

6.3.2 Rate

A rate, like a ratio, is also a comparison between two numbers or measurements, but the two numbers in a rate have different units.

Some examples of rate include cost rates, (for example food item cost RO16,95 per kg or 16,95 RO/kg) and speed (for example, a car travels at 60km/h).

When we calculate rate, we divide by the second value, so we are finding the amount per one unit.

Unit rates

For example, we have RO 20 to buy 2 kg of gun powder, we write:
$RO20 : 2\,kg = RO10 : 1\,kg = RO\ 10/kg.$

This rate is a unit rate. Thus 10 Rial Omani per 1 kg.

Example: Elias, a star athlete runs 100m in 15seconds. What is his speed in metres per second?

Solution: $100\,m \div 15\,sec = 6.67 m/sec$

6.3.3 Proportion

A proportion is a statement showing equality of two ratios, such as $a : b = c : d$

$$or \quad \frac{a}{b} = \frac{c}{d}$$

where b and d are not zeros. From this relation we arrive at: $ad = bc$.

Example: The ratio of teachers to students in one school is 1:20. There are 300 students. How many teachers are there?

Solution: Given ratio is $\frac{teacher}{student} = \frac{1}{20}$

Let the number of teachers be = x when there are 300 students, then $\frac{1}{20} = \frac{x}{300}$ or

$$x = \frac{300}{20} = 15$$

thus for 300 students there are 15 teachers.

6.3.4 Direct Proportion

Two quantities are in **direct proportion**, if they increase or decrease at the same **rate**.

Direct proportion or direct variation is the relation between two quantities where the ratio of the two is equal to a constant value.

Example 1) If 1 pen cost OMR 0.500, then 2 pens cost OMR 1, 3 pens cost OMR 1.500 and so on.

In the example shown above, if C is the cost of pens and N is the number of pens bought, we write: $C \propto N$ (C is proportional to N)

or, $C = kN$, where 'k' is called the constant of proportionality.

Example 2:
If p is proportional to q and $p = 3$ when $q = 12$, then find the following:

(a) The value of p when $q = 30$
(b) The value of q when $p = 18$

Solution: p is proportional to q or $p \propto q$
$\Rightarrow p = kq$

$$\Rightarrow k = \frac{p}{q}$$

$$\Rightarrow k = \frac{3}{12} \ (p = 3 \text{ when } q = 12)$$

$$\Rightarrow k = 0.25$$

$$\therefore p = 0.25q$$

(a) $p = 0.25 \times 30 = \mathbf{7.5}$

$(b)\ q = \frac{p}{0.25} = \frac{18}{0.25} = \mathbf{72}$

Class Activity 6.3.2

1. A car uses 4 litres for every 48km distance travelled. How many litres will it use to cover a distance of 285km?

2. y is directly proportional to x. When $y = 24, x = 8,$ find the value of constant 'k'. What is the value of y when $x = 2$?

3. 12 students need to drink a total of 3 litres of coffee over the course of an activity. How much coffee would be needed for 20 students during the same activity? Give your answer in litres.

4. In a certain apartment building, the ratio of tenants to cars is 3:5. If there are 30 tenants, How many cars are there?

5. If $\frac{x}{9} = \frac{2}{3}$, find x.

6.3.5 Inverse Proportion

Two quantities are in inverse proportion, if as one increases, the other decreases or if one decreases, the other increases.
The product of the two quantities is constant.

Example 3: 12 men can do a piece of work in 50 days. How many men would be required to do the work in 30 days?
Solution:
Let x be the number of men required.
$12(50) = x(30)$
$x = \frac{12\,(50)}{30} = 20\, men$

Method: Remember: Strategy for solving proportion problems
1. Find the value of k (the constant of proportionality).
2. Use k together with given values to answer the question.

Example 4: Boyle's Law states that at constant temperature, the volume V of a gas is inversely proportional to its pressure P. When the pressure is $600\, N/m^2$, the volume is $4m^3$.
Find:
(a) The volume when the pressure is $400\, N/m^2$
(b) The pressure when the volume of the gas is $5\, m^3$

Solution: $V \propto \frac{1}{P}$ or $V = \frac{k}{P}$ **or** $k = VP$
$k = 4 \times 600 = 2400$
(a) The volume when the pressure is $400\, N/m2$
$V = \frac{k}{P} = \frac{2400}{400} = 6m^3$

(b) The pressure when the volume of the gas is 5 m^3
$V = \frac{k}{P}$ *or* $P = \frac{k}{V} = \frac{2400}{5} = 480 N/m^2$

Link: Direct and Inverse Proportion

Class Activity 6.3.3

1) If x is inversely proportional to y, Given that when $x = 3, y = 4$, find x when $y = 6$.

2) If 15 machines can do a piece of work in 5 days. How many machines of the same type would be required to do the same work in 3 days, assuming they work at the same rate?

3) A car will take a time of 2 hours to cover a distance of x km when travelling at a speed of $100km/h$. How many hours will the same car take if it travels at a speed of $120km/h$?

4) It takes 22 hours for a hosepipe with a flow of 12 litres per minute to fill a swimming pool. How long will it take if its flow is reduced to 5 litres per minute?

5) The time taken to build a house is inversely proportional to the number of builders. If there are 6 builders, it takes 80 days to complete the house. How many builders must be employed to build the house in just 16 days?

6.4 Map Scales

Scale Drawings are a good example of the application of direct proportion in everyday life engineering designs.

Scale Drawings

The map of Oman

A map drawn to a scale of $\mathbf{1:100\,000}$ means that the real distance is 100 000 times the length of 1 unit on the map or drawing.

A scale is usually expressed in one of two ways:

a) Using units as in $1\ cm\ to\ 1\ km$
or

b) without explicitly mentioning units as in $1:100\,000$.

Example 1: Write the scale $1\ m\ to\ 1\ km$ in ratio form.

Solution: $1m\ to\ 1\ km\ =\ 1m:1km$

$$= 1m:1000m$$
$$= 1:1000$$

Example 2: Simplify the scale 5 cm: 2 km

Solution: $5cm\ to\ 2\ km\ =\ 5cm:2000m$

$$= 5cm:200\,000cm$$
$$= 5:200\,000$$
$$= 1:40\,000$$

Calculating the Actual Distance using Scale

If the scale is 1: $\boldsymbol{x}$, then multiply the map distance by x to calculate the actual distance.

Example 3: A particular map shows a scale of $1:5000$. What is the actual distance if the map distance is 8 cm?

Solution: $Scale\ =\ 1:5000\ =\ 1cm:5000cm$

$$\therefore \text{Map distance}:\text{Actual distance} = 1:5000$$
$$= 1\times 8:5000\times 8$$
$$= 8cm:40\,000cm$$

Therefore, the distance of 8cm is representing a distance $\boldsymbol{of\ \ 40\,000cm\ or\ 400m}$ on actual ground.

Class Activity 6.4

1) Simplify the scale $5mm:1m$.

2) Simplify the scale $5cm:2km$.

3) A map has a scale of $1:200000$. The distance between two towns is $60km$. How far apart are the towns on the map?

Worksheet-6 (a)

Circle the correct answer in the following questions (1-6):

1) $2.64\, megavolts\ =\ \ldots\ldots\ldots\ldots\ldots$kilovolts

   (a) 2640 (b) 0.00264 (c) 26400

2) If 1 inch $= 2.54\, cm$ and 1 $cm\ =\ 10mm$ then 6 mm is equal to how many inches?

   (a) 0.625" (b) 0.236" (c) 0.375"

3) $45^{\circ} F =$ _________K is

   (a) -280.37 (b) 280.37 (c) 296.55

4) The largest distance between 20 miles, 30 km and 30000ft is

   (a) 20 miles (b) 30km (c) 30000ft

5) The following is the table of three cities with temperature on a particular day.

City	Temperature
Muscat	$32°C$
London	$72°F$
New York.	310.15 k

   The city with the maximum temperature is

   (a) Muscat (b) London (c) New York

6) The temperature dropped 15 degree Celsius in the last 30 days. If the rate of temperature drop remains the same, how many degrees will the temperature drop in the next ten days?

   (a) 5 (b) 10 (c) 12

7) Convert 45 millimeter (mm) into the equivalent length in centimeters (cm)

8) Convert 50 $km/hour$ into m/sec.

Show all your working step by step for the following questions (9 – 16)

9) A car is driven for 4500000 cm. What is the distance in kilometres?

10) A tank contains 40000 litres of liquid. How many cubic metres does it contain?

11) Convert the following to Fahrenheit
    $37^0\ C =$ ______F

12) Convert the following to Celsius
    $45^0\ F =$ ______C

13) Convert the following to Kelvin
    $-50^0\ C =$ ______K

14) Convert the following to Fahrenheit
    $100\ K =$ ______F

15) A particular map shows a scale of 1:10000. What is the actual distance if the map distance is 15 cm?

16) A model of a car has a scale of $3cm : 2m$. the length of the actual hood of the car is $1m$. What is the length of the hood of the model?

Worksheet-6(b)

Circle the correct answer in the questions 1 to 10:

1. 8 marks out of 20 as a percentage is equal to:

   (a) 45 % (b) 40 % (c) 0.45 %

2. Express 1200 m as a percentage of 3 km

   (a) 36 % (b) 400 % (c) 40 %

3. A college team won 9 games this year against 6 games won last year. What is the per cent increase?

   (a) 50 % (b) 40 % (c) 25 %

4. A student has to obtain 60% of the total marks to pass. He got 250 marks and failed by 50 marks. The maximum marks are:

   (a) 500 (b) 300 (c) 400

5. The ratio of 6 : 4 can also be expressed as

   (a) 64 % (b) 66 % (c) 150 %

6. If angles of a triangle are in the ratio 2 : 3 : 4. The least of the three angles is

   (a) 40° (b) 60° (c) 80°

7. The selling price of a toy car 5 OMR. If the profit made by shopkeeper is 25%, what is the cost price of this toy?

   (a) 4 OMR (b) 3 OMR (c) 4.5 OMR

8. A school has 8 periods a day each of 45 minutes duration. How long would each period be, if the school has 9 periods a day, assuming the number of school hours to be the same?

   (a) 40 min (b) 35 min (c) 30 min

9. 10 pipes are required to fill a tank in 1 hour 40 minutes. How long will it take if only 8 pipes of the same type are used?

   (a) $2h\ 5min$ (b) $4h\ 5min$ (c) $1h\ 25min$

10. If $x : y = 3 : 2$, the value of $(2x + 3y) : (3x + y)$ is

    (a) $12 : 11$ (b) $11 : 12$ (c) $10 : 11$

Show clearly all necessary working steps.
Give the answer up to 2 decimal places if your answer is not exact.

11. A student attended 49 hours out of 70 hours. What is his percentage % attendance?

12. In a survey of 420 pupils it was found that 70% had personal cars. How many pupils had personal cars?

13. 20% of a certain length is 50cm. What is the complete length?

14. If an amount of 32 RO increased by 25%. Find the new amount**.**

15. If **15** machines can manufacture certain products in 20 days. Assuming that they work at the same rate, how many machines of the same type would be required to produce the same number of products in 60 days?

(Unit-7) Linear equations and inequalities

7.1 Solving linear equations

Any equation of the form: $ax + b = 0, \ \ a \neq 0$, where a and b are real constants and x is a variable, is called a linear equation in one variable.

The solution of linear equation $ax + b = 0$ is $\frac{-b}{a}$, which means that if we substitute $x = \frac{-b}{a}$ in

$ax + b = 0$, we will get $0 = 0$ (LHS=RHS)

(i.e. Left hand side = Right hand side)

Examples of linear equations:

1) $5x - 10 = 0$
2) $4(x + 2) - 6x = 4$

Example 1: Solve $5x - 6 = 4$

Solution: $5x - 6 + 6 = 4 + 6$

$$5x = 10$$

$$\frac{5x}{5} = \frac{10}{5}$$

$$x = 2$$

Example 2: Solve $5x - 9 = 3x + 7$ and check.

Solution: $5x - 9 + 9 = 3x + 7 + 9$

$$5x = 3x + 16$$

$$5x - 3x = 3x + 16 - 3x$$

$$2x = 16$$

$$\frac{2x}{2} = \frac{16}{2}$$

$$x = 8$$

Check:

$5x - 9 = 3x + 7$ Original equation

$5(8) - 9 = 3(8) + 7$ Substitute $x = 8$

$40 - 9 = 24 + 7$

$31 = 31$ (R.H.S=L.H.S, a true statement)

Class Activity 7.1

Solve for the variable in each linear equation:

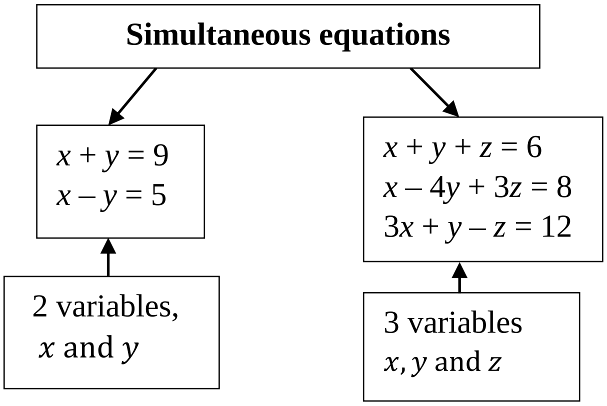
1. $4(x + 2) - 6x = 4$

2. $\frac{x}{2} = \frac{x+1}{3}$

3. $5 - \frac{3a-4}{5} = \frac{6-2a}{2}$

7.2 Simultaneous linear equations

Simultaneous equations are a number of equations which are true at the same time for a given set of values of the variables.



However at this level we shall focus on simultaneous linear equations with only 2 variables.

There are 3 main ways to solve simultaneous linear equations and these are:

- Solution by elimination method
- Solution by substitution method
- Solution by graphical method

7.2.1 Solution by Elimination Method

Steps to solve simultaneous equations by Elimination: "O M E S S"

1) **O**rganize your equations and label them

2) **M**ake the two coefficients the same- by multiplying with constant(s) **if they are not the same.**

3) **E**liminate one variable-
Same signs → subtract the equations
Alternate signs →add the equations

4) **S**olve the equation

5) **S**ubstitute the value obtained into any of the two equations to get the other unknown variable.

Example 1: Solve $x + y = 9$
$x - y = 5$

Solution:

Step 1) Label the equations (1) and (2)
$x + y = 9$ ....(1)
$x - y = 5$......... (2)

Step 2) Compare the coefficients of x and y in both equations. The coefficients of x are the same but those of y are different.

Step 3) Add the two equations (1) and (2) to eliminate 'y'

Step 4) We have $2x = 14$
$x = 7$

Step 5) Substituting the value of x in the equation (1), gives, $7 + y = 9$
$y = 9 - 7 = 2$,

Thus the solutions are $x = 7, y = 2$

Example 2: Solve for a and b;
$2a = 16 - 5b$
$a + b = 5$

Solution:

The above equations can be re-written as
$2a + 5b = 16$…………..(1)
$a + b = 5$…………… (2)

Multiply the equation (2) by 2 to get equation (3); → $2a + 2b = 10$………......(3)

Re-writing equations (1) and (3) we have
$2a + 5b = 16$…………..(1)
$2a + 2b = 10$…………..(3)
Subtract (3) from (1) to eliminate 'a'
we get $3b = 6$, so, $\boldsymbol{b = 2}$.
We can substitute the value $b = 2$ in equation (2) [or (1)] to get $a + 2 = 5$, hence $\boldsymbol{a = 3}$.

Link:Solution by Elimination

Class Activity 7.2.1

Solve the following simultaneous equations:

1) $3x + y = 2$
   $6x - y = 25$

2) $x + y = 8$
   $3x + 2y = 21$

3) $x + y = 4$
   $2x - 3y = 18$

4) $2a - 3b = 5$
   $3a - 2b = 20$

7.2.2 Solution by Substitution

Steps in solving simultaneous equations by Substitution method:

1) Make one of the variables the subject from any of the two equations.
2) Substitute the variable in the other equation so that you deal with an equation having one unknown variable.
3) Substitute the value of the variable in the equation where the other variable is the subject to get the remaining unknown value.

Example 1: Solve $x + y = 9$
$x - y = 5$

Solution:

Step 1) Label or name the equations (1) and (2)

$x + y = 9$……….(1)
$x - y = 5$………(2)

From equation (2) we have
$x = y + 5$……….(3)

Step 2) We substitute x in (1), then we have
$(y + 5) + y = 9$, all in terms of y
$2y + 5 = 9$
$2y = 9 - 5 = 4$
$y = 2.$

Step 3) Substitute the value of y in (3);
$x = y + 5$, then $x = 2 + 5 \Rightarrow x = 7.$

Hence $\boldsymbol{x = 7}$ and $\boldsymbol{y = 2}$

Link: Solution by Substitution

Class Activity 7.2.2

Using the substitution method solve

1) $y + 1 = 2x$
$y = x + 2$

2) $y = 2x + 4$
$3x + y = 9$

3) $x + y = 3$
$2x + 3y = 8$

4) $2x + 5y = 12$
$4x - y = 2$

5) $3x + 5y = -2$
$2x - y = 3$

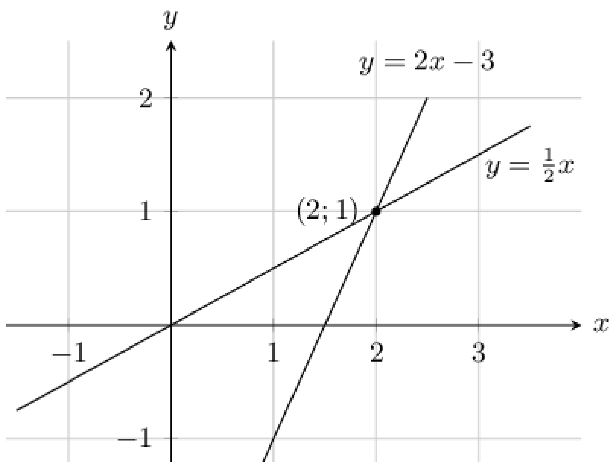
7.2.3 Solution by graphical method

If the graphs of each linear equation are drawn, then the solution to the system of simultaneous equations is the coordinates of the point at which the two graphs intersect.

For **example:** $x = 2y$ and $y = 2x - 3$

To represent the graphs of the two equations we need to have them re-written as

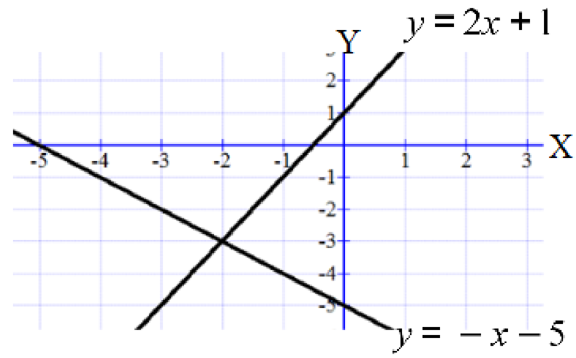
$x = 2y \rightarrow y = \frac{1}{2}x$ and $y = 2x - 3$



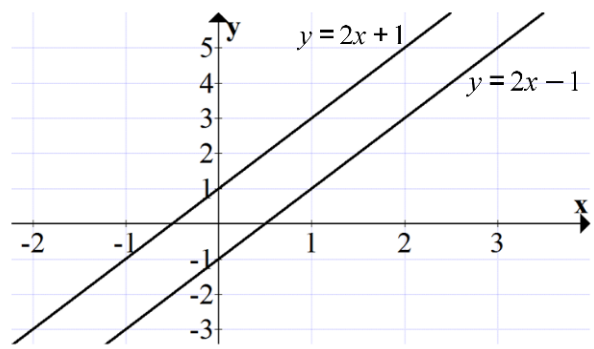
Since the graphs of the two linear equations meet at the point (2,1), $\therefore x = 2 \; and \; y = 1$
are the solutions of the simultaneous equations.

Class Activity 7.2.3

1) Look at the graph below and write down the solutions for the simultaneous equations:
   $y = 2x + 1$
   $y = -x - 5$



2) The following simultaneous equations have no solutions
   a) True
   b) False



7.3 Linear inequalities

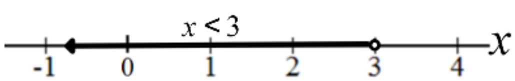
Any inequality of the form:

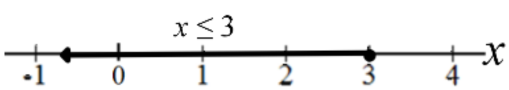
$ax + b > 0$, $ax + b \geq 0$, $ax + b < 0$, or $ax + b \leq 0$; etc where a and b are real constants, with $a \neq 0$ and x is a variable, is called a linear inequality in one variable.

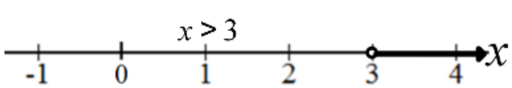
Table for symbols of inequalities

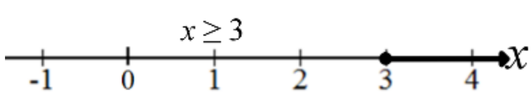
Symbol	Meaning	Example
$<$	less than	$x < 3$
$\leq$	less than or equal to	$x \leq 3$
$>$	greater than	$x > 3$
$\geq$	greater than or equal to	$x \geq 3$
$\neq$	not equal to	$x \neq 3$

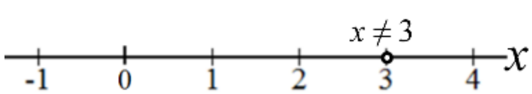
Graphical representation of examples given above





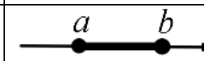
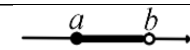
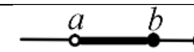
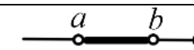
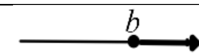
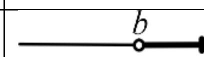
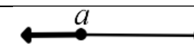
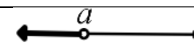






7.3.1 Methods of describing inequalities

Note:

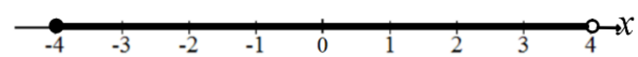
Type	**Inequality notation**	**Line graph**	**Interval notation**
Closed	$a \leq x \leq b$		$[a, b]$
Half Open	$a \leq x < b$		$[a, b)$
Half Open	$a < x \leq b$		$(a, b]$
Open	$a < x < b$		(a, b)
Closed	$x \geq b$		$[b, \infty)$
Open	$x > b$		(b, ∞)
Closed	$x \leq a$		$(-\infty, a]$
Open	$x < a$		$(-\infty, a)$

1. In above table, $\boldsymbol{a}$ is called left end point and $\boldsymbol{b}$ is called right end point or in general end points.
2. ∞ is read as **infinity**.
3. $(-\infty, \infty)$ represents real number line.

Example 1: Write the following in inequality notation and graph on a real number line: $[-4, 4)$.
Solution: Interval notation: $[-4,\ 4)$

Inequality notation: $-4 \leq x < 4$

Line graph:

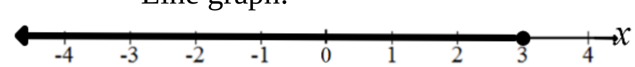


Example 2: Write the following in inequality notation and graph on a real number line: $(-\infty,\ 3]$

Solution: Interval notation: $(-\infty,\ 3]$

Inequality notation: $x \leq 3$

Line graph:



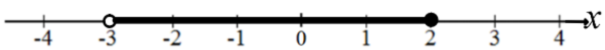
Example 3: Write the following in interval notation and graph on a real number line.

$$-3 < x \leq 2$$

Solution: Interval notation: $(-3, \ 2]$

Inequality notation: $-3 < x \leq 2$

Line graph:



Class Activity 7.3.1

1. Write each of the following in inequality notation and draw the graph on a real **number line.**

Interval	**Inequality**	**Graph**
(i) $[-2, \ 3)$		
(ii) $(-4, 2)$		
(iii) $[-2, \infty)$		
(iv) $(-\infty, 3\ )$		

2. Write each of the following in interval notation and draw the graph on a real number line.

Interval	**Inequality**	**Graph**
i)	$-2 < x \leq 6$	
ii)	$-5 \leq x \leq 5$	
iii)	$-7 < x < 8$	
iv)	$-4 \leq x < 5$	
v)	$x \leq -2$	
vi)	$x > 3$	

7.3.2 Solving linear inequalities

The following rules apply when solving inequalities

(i) The rules for solving inequalities are the same as those used when solving equations.
- Thus if you add or subtract a value from the left side of the inequality you should do the same to the right side of the inequality for the inequality to remain true.

(ii) If we divide or multiply both sides of an inequality by any number with a minus sign, the direction of the inequality changes!

$$-1 < 4 \Rightarrow 1 > -4$$

The examples which follow will show you how the rules i) and ii) apply.

Example 4: Solve $3x - 5 < 7$

Solution: $3x - 5 < 7$

$$3x - 5 + 5 < 7 + 5$$

$$3x < 12$$

$$\frac{3x}{3} < \frac{12}{3}$$

$$x < 4$$

Example 5: Solve linear inequality

$$\frac{5}{3}(4x - 7) > 2$$

Solution: $\frac{5}{3}(4x - 7) > 2$

$$3 \times \frac{5}{3}(4x - 7) > 3 \times 2$$

$$5(4x - 7) > 6$$

$$20x - 35 > 6$$

$$20x > 41$$

$$x > \frac{41}{20}$$

Note: In the examples 4 and 5, above the rule i) is applied.

Now let us see how the rule (ii) is used.

Example 6: Solve the inequality $-2x < 8$

Solution:
We have to divide both sides by -2,but in this case if we leave the answer as $x < -4$, it will be false since if we started with a simple inequality of simple numbers like $-2 < 8$, which is true then if decide to divide or multiply both sides by -1, we would get $2 < -8$!, **which is definitely not true**. Hence the need to change the inequality sign to face the other side as $2 > -8$
So the solution for the inequality $-2x < 8$ is
$\boldsymbol{x > -4}$**.**
Example 7: Solve and graph: $-3 \le 4 - 7x < 18$

Solution: $-3 \le 4 - 7x < 18$

$$-3 - 4 \le 4 - 7x - 4 < 18 - 4$$

$$-7 \le -7x < 14$$

$$\frac{-7}{-7} \ge \frac{-7x}{-7} > \frac{14}{-7}$$

$1 \ge x > -2$ same as $-2 < x \le 1$
or $(-2,1]$

Graphically

Class Activity 7.3.2

1. Solve the following linear inequalities.

(i) $4x - 5 < 9$

(ii) $5(2x - 1) > 15$

(iii) $\frac{7}{2}(5x - 6) > 14$

(iv) $4(3x - 2) < 2(x + 6)$

(v) $-3x > 6$

(vi) $\frac{x}{-3} < 4$

2. Solve and graph: $-3 < 7 - 2x \le 7$

Link: Solving linear Inequalities in one variable

Worksheet-7

Solve the following linear equations. (Questions 1-6)

1. $3(y-4)+2y=18$

2. $\frac{1}{2}x+0.25x=\frac{15}{2}$

3. $4t-3(t+2)+t=5(t-1)-7t$

4. $\frac{x}{3}-\frac{x}{5}=2$

5. $\frac{x+3}{4}=\frac{x-3}{5}+2$

6. $0.6x+\frac{4}{5}=0.28x+1.16$

Solve the following simultaneous linear equations (Questions 7 – 9):

7. $x+y=7$
   $2x+3y=3$

8. $2x-t=5$
   $3x+4t=2$

9. $3x+2y=7$
   $2x+3y=3$

10. Rewrite the following in inequality notation and show the graph on a real number line.

(i) [-2, 3]

(ii) (-2, 3)

(iii) [-2, 3)

(iv) (-2, 3]

(v) [-2, ∞)

(vi) (-∞, 3]

2) Rewrite the following in interval notation and draw the graph on a real number line.

(i) $-3 < x \leq 4$

(ii) $-3 \leq x \leq 4$

(iii) $-3 \leq x < 4$

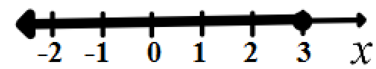
(iv) $-3 < x < 4$

(v) $x < -3$

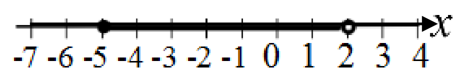
(vi) $x \geq 4$

11. Write the following in interval and inequality notation.

(i)



(ii)



12. Solve and graph the solution of the following linear inequalities.

(i) $2(2x+3)-10 < 6(x-2)$

(ii) $-4 < x + 2 < 8$

(iii) $2 \leq -3x - 1 \leq 5$

(iv) $\frac{9x}{5} + 1 > \frac{7x}{15} + 5$

(v) $-3 \leq \frac{x-1}{-2} < 4$

(Unit 8) Modelling simple real life problems

8.1 Word problems on Equations

In this section, formulation and solution of some practical problems will be discussed. These problems involve relations among unknown quantity (variable) and known quantities (numbers). We often refer to these problems as word problems.

The following general approach to solving word problems should be followed.

1. Read the problem carefully and figure out what it is asking you to find.

Usually, but not always, you can find this information at the end of the problem.

2. Assign a variable to the quantity you are trying to find.

Most people choose to use x, but feel free to use any variable you like. For example, if you are being asked to find a number, some students like to use the variable n. It is your choice.

3. Write down what the variable represents.

At the time you decide what the variable will represent, you may think there is no need to write that down in words. However, by the time you read the problem several more times and solve the equation, it is easy to forget where you started.

4. Re-read the problem and write an equation for the quantities given in the problem.

This is where most students feel they have the most trouble. The only way to truly master this step is through lots of practice. Be prepared to do a lot of problems.

5. Solve the equation.

The examples done in this section/lesson will be linear equations.

6. Answer the question in the problem!

Just because you found an answer to your equation does not necessarily mean you are finished with the problem. Many times you will need to take the answer you get from the equation and use it in some other way to answer the question originally given in the problem.

7. Check your solution.

Your answer should not only make sense logically, but it should also make the equation true. If you are asked for a time value and end up with a negative number, this should indicate that you've made an error somewhere. If you are asked how fast a person is running and give an answer of 700 km/h, again you should be worried that there is an error. If you substitute these unreasonable answers into the equation you used in step 4 and it makes the equation true, then you should re-think the validity of your equation.

Consecutive Numbers

Consecutive means coming one after another in the order from smallest to largest.
For example:

1, 2, 3, 4, 5, 6, and so on are consecutive numbers.

Examples of consecutive even numbers are:
0, 2, 4, 6, 8, 10, 12...

Examples of consecutive odd numbers are:
1, 3, 5, 7, 9, 11, 13, 15, ...

Example 1: The sum of two consecutive numbers is 53. Find the numbers.

Solution:

Let the numbers be x and $x+1$.

$$x+(x+1)=53$$
$$2x+1=53$$
$$2x=52$$
$$x=26$$

∴The numbers represented *by* x and $x+1$ are **26** and **27.**

Alternatively we could have assumed the numbers to be x and $x-1$

then the equation would be:

$$x+(x-1)=53$$
$$2x-1=53$$
$$2x=54$$
$$x=27$$

Still the numbers x and $x-1$ will be **27** and **26**

Example 2: If 7 is subtracted from five times a number, the result is 63. Find the number.

Solution:

Let the number be x. *(assigning a variable to the unknown quantity)*

$5x-7=63$ *(formation of equation)*

$5x=70$

$x=14$

∴ The number is 14.*(Answer to the question)*

Example 3: The sum of two numbers is 25. One of the numbers exceeds the other by 9. Find the numbers.

Solution:

Let the number be x.

Then the other number is $(x+9)$.

$$x+(x+9)=25$$
$$2x+9=25$$
$$2x=16$$
$$x=8$$

∴The numbers are **8** and $8+9=$ **17**.

Alternatively let the numbers be x and $(x-9)$

Then the equation is $x+(x-9)=25$

$$2x-9=25$$
$$2x=34$$
$$x=17$$

Hence the numbers are **17** and $17-9=$ **8**

Which is the same as the result previously obtained.

Link1: Problem 1 involving linear equations

Link 2: Problem 1 involving linear equations

Class Activity 8.1

1) Twice a number added to $\frac{3}{4}$ of the number gives 55. Find the number.

2) Find two consecutive odd integers whose sum is -36.

3) In a Math test, the highest grade was 42 points higher than the lowest grade. The sum of the two grades was 138. Find the lowest grade.

4) In a given amount of time, Salim drove twice as far as Khalid. Altogether they drove 90 kilometers. Find the number of kilometers driven by each.

8.2 Word problems on Inequalities

In Unit 6.3, we dealt with **Inequalities** as the relationships between two expressions which are not equal to one another. Symbols such as $<, >, \leq, \geq$, were used to express such relationships, where as in equations we used the " $=$ " equal sign.

Word problems leading to or involving inequalities are dealt with in the same way as we did in word problems on equations. The only difference is that we use any of the symbols previously mentioned " $<, >, \leq, \geq$ " and express or solve the relationships.

Some key words and symbols to remember are shown below again for you to use when forming the inequalities.

Key Word	**Symbol**	**Example**
Less than	$<$	$3 < 5$, (3 is less than 5)
Less than or equal to	$\leq$	$3 \leq 5$ (3 is less than or equal to 5)
Greater than	$>$	$6 > 1$ (6 is greater than 1)
Greater than or equal to	$\geq$	$2 \geq 2$ (2 is greater than or equal to 2)
At most, not more than or maximum of	$\leq$	$x \leq 10$ x is not more than 10
At least, not less than or minimum of	$\geq$	$x \geq 10$ x is at least 10

Example 1:

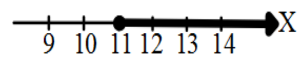
6 is added to a number x. Then the result is multiplied by 9. The final answer is greater than 99. Form an inequality to show the situation

Answer: $9(x+6) > 99$

Example 2: Form the inequality, solve and show on a graph**:**

7 is subtracted from a number y. Then the result is multiplied by 3. The final answer is greater than or equal to 12.

Solution:
$$3(y-7) \geq 12$$
$$3y - 21 \geq 12$$
$$3y \geq 33$$
$$y \geq 11$$

Graph: 

Example 3: Find all pairs of consecutive odd positive integers, both of which are smaller than 18, such that their sum is more than 20.

Solution : Let x be the smaller of the two consecutive odd positive integers.

Then, the other odd integer is $(x + 2)$.

Given : Both the integers are smaller than 18.

In solving this kind of problems, when both the smaller and larger integers are less than 18, always we have to take the larger integer to form inequality.

Then, we have: $x + 2 < 18$

$$\Rightarrow x < 16 ------(1)$$

Given : Sum of the integers is more than 20.

Then, we have: $x + (x + 2) > 20$

$$2x + 2 > 20$$
$$2x > 18$$
$$x > 9 \Rightarrow 9 < x ------(2)$$

Combine (1) and (2).

$$9 < x < 16$$

So, the value of x is any odd integer between 9 and 16.

∴The possible values of x are 11, 13, 15

When $x = 11, 13, 15$, the possibles values of $(x + 2)$ are 13, 15, 17

Therefore the required pairs of odd integers are **(11, 13), (13, 15)** and **(15, 17)**

Class Activity 8.2

1. **Show using an inequality**: The sum of two consecutive numbers is not more than 20.

2. **Form, solve the inequality and draw the graph**:
   Half of a number is greater than 12.

3. The sum of two positive numbers should not be less than 10 and not be more than 20. If one number is $\boldsymbol{x}$ and the other is 6 more than this, what are the possible values of $\boldsymbol{x}$?

4. **Form the inequality and solve and draw the graph of your answer:**
   A number x is multiplied by 4, then 8 is subtracted from the result. The final answer is less than or equal to 28.

Worksheet 8

(1) The ratio of two numbers is 3:5 and their difference is 16. What are the numbers?
 a) 24 and 40
 b) 20 and 36
 c) 8 and 24

(2) The sum of two numbers is 75. The bigger number is 15 more than three times the smaller number. What is the bigger number?
 a) 15
 b) 60
 c) 45

(3) At coffee shop, Fatima wants to order some pieces of donuts and a cup of coffee. She plans to spend no more than 4 OMR. If 1 piece of donut costs 0.500 OMR and a cup of coffee costs 1.000 OMR. Taking n as the number of donuts she wants to eat, which of the following inequalities describes her bill?
 a) $0.5n + 1 < 4$
 b) $0.5n + 1 \leq 4$
 c) $0.5n + 1 \geq 4$

(4) Saif is a salesman in a medical supplies company and he receives a regular salary of 500 OMR plus 10% of his sales amount every month. What amount of sales should he have to give him monthly income twice his salary or more?
 a) at most 1000 OMR
 b) not less than 5000 OMR
 c) between 1000 OMR and 5000 OMR

(5) Male and female students are to sit in a classroom with 30 arm chairs available but a chair in between should be left vacant. If there are at most 5 females to be in the room, how many male students could possibly sit in the room?
 a) 10 to 14
 b) at least 15
 c) less than 10

Show all your working step by step for the following questions (6 – 15)

(6) When 6 is added to four times a number, the results is 50. Find the number.

(7) The cost of three tables and five chairs is 150 OMR. If the table costs 18 OMR more than the chair, find the cost of the table and the chair.

(8) There are 300 students in a school. If the number of boys is 60 more than the girls, how many boys are there in the school?

(9) If 3 less than 4 times a number x is between 7 and 9, what is the range of the possible values of x?

(10) One side of a triangle is 4m longer than the shortest side and another is 3m longer than the shortest as well. If the perimeter of the triangle is $25m$, find the lengths of the three sides of the triangle.

(11) The three angles of a triangle are x^o, $(x+30)^o$ and $(x-6)^o$. Find the size of each angle of the triangle.

(12) Three tables and five chairs together cost OMR 9.900. If a table costs twice as much as a chair, find the cost of one table and one chair.

(13) The length of a rectangle is 5 cm less than twice its width. If the perimeter of the rectangle is no more than 80cm, what are the maximum possible dimensions?

(14) The sum of a number and 80 is greater than the product of – 3 and that number. What are the possible values for the number?

(15) A man wants to cut three lengths from a single piece of board of length 91 cm. The second length is to be 3 cm longer than the shortest and third length is to be twice as the shortest. What are the possible lengths for the shortest piece, if third piece is to be at least 5 cm longer than the second?

(16) Find three consecutive odd integers such that the sum of 7 times the smallest and twice the largest is -91 .

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