Military Technological College

BASIC MATHEMATICS

WORKBOOK-1 **MODULE CODE: MTCG1016**

Term-1 **AY:2023-24** **FPD-MATH1**

MILITARY TECHNOLOGICAL COLLEGE

Delivery Plan - Year 2023-24 [Term 1]

Title / Module Code / Programme	Basic Mathematics /MTCG1016/Foundation Programme Department (FPD)	Module Coordinator	Rajendar Palli
Lecturers	TBA	Resources & Reference books	Moodle & Workbook
Duration & Contact Hours	Term 1: 5 hrs x 11 weeks = 55 hours		

WEEK No.	TOPICS	HOURS	LEARNING OUTCOMES
1	**Introduction, Delivery of Material etc.**	1	1,2
	Basic Theory of Numbers & Operations 1.1 Basic Theory of Numbers 1.2 Arithmetic Operations and Fundamental Laws 1.2.1 The Four Basic Operations **Online Quiz 1** 1.2.2 Fundamental Laws of Operations 1.3 Directed Numbers and their Operations 1.4 Sequence of Arithmetic Operations **Online Quiz 2** **Worksheet 1-Moodle Online Test 1**	3	
	Set Theory 2.1 Sets, Types of Sets, Subsets, Venn Diagrams **Online Quiz 3**	1	
2	2.2 Union and intersection of Sets **Online Quiz 4** 2.3 Application of Set Theory Worksheet 2-Moodle Online Test 2	2	2, 3 and 8
	Basic Arithmetic 3.1 Factors and Multiples of a Number 3.1.1 Highest Common Factor (HCF) **Online Quiz 5** 3.1.2 Lowest Common Multiple (LCM) 3.1.3 Application of HCF & LCM **Online Quiz 6** 3.2 Reducing fractions to Simplest form 3.3 Addition and Subtraction of Fractions **Online Quiz 7**	3	

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Online Quiz 8
3.5 Estimation/Rounding off
3.6 Scientific Notation
Online Quiz 9 & Worksheet 3- Moodle Online Test 3</td><td>2</td><td rowspan="3">3, 6 and 8</td></tr>
<tr><td>Basic algebra (Part-1)
4.1 Power Number Algebra and Laws of Indices
Online Quiz 10
4.2 Algebra- Use of Symbols & Substitution
4.3. Addition and Subtraction of Polynomials
4.4 Multiplication of Polynomials
Online Quiz 11
Worksheet 4 – Moodle Online Test 4</td><td>3</td></tr>
<tr><td>Revision for CA1</td><td></td></tr>
<tr><td>Continuous Assessment 1(Topics: Units 1, 2 and 3)</td><td></td><td>1,2,3,6,8</td></tr>
<tr><td rowspan="2">4</td><td>Techniques of Factorization and Rational Expressions
5.1 Factorisation of Polynomials
Online Quiz 12
5.2 Simplification of Rational Expressions
5.3 Multiplication and Division of Rational Expressions
Online Quiz 13
5.4 Addition and Subtraction of Rational Fractions
5.5 Rationalising denominators of Irrational Expressions
Online Quiz 14
Worksheet 5 – Moodle Online Test 5</td><td>4</td><td rowspan="2">3, 4, 6, 8</td></tr>
<tr><td>Units of Measurements, Percentages and Ratios
6.1 and 6.1.1 Units of Measurements and Conversions
Online Quiz 15</td><td>1</td></tr>
<tr><td rowspan="2">5</td><td>6.1.2 Inter-system conversions
6.1.3 Measuring Temperature
Online Quiz 16
6.2 Percentages
Online Quiz 17
6.3 Ratio and Proportion
6.3.1 Ratio and 6.3.2 Rate
Online Quiz 18
6.3.3 and 6.3.4 Direct Proportion and Inverse Proportion
6.4 Map Scales (Online Quiz 19)
Worksheet 6-Moodle Online Test 6</td><td>4</td><td rowspan="2">3,5,8</td></tr>
<tr><td>Linear Equations, Inequalities and their Applications
7.1 Solving Linear Equations</td><td>1</td></tr>
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WEEK No.	TOPICS	HOURS	LEARNING OUTCOMES
6	7.2 Simultaneous Linear Equations 7.2.1 Solution by Elimination Method **Online Quiz 20** 7.2.2 Solution by Substitution Method 7.3 Linear Inequalities 7.3.1 Methods of describing inequalities **(Online Quiz 21)** 7.3.2 Solving linear inequalities **(Online Quiz 22)** **Worksheet 7 – Moodle Online Test 7**	3	5, 9
	Modelling Simple Real Life Problems 8.1 Word problems on linear equations 8.2 Word problems on linear Inequalities **Online Quiz 23** **Worksheet 8 – Moodle Online Test 8**	2	
7	**Quadratic Equations and Formulas** 9.1 Solving Quadratic equations 9.1.1 and 9.1.2 Solutions by Factorisation and Formula **Online Quiz 24** 9.2 Formation of Quadratic Equations 9.3 Equations involving radicals **(Online Quiz 25)** 9.4 Formula Transposition/subject change in Formula **Online Quiz 26** **Worksheet 9-Moodle Online Test 9**	3	7
	Revision of Units 4 to 8		
	Continuous Assessment 2(Topics: Units 4, 5, 6, 7 and 8)	2	3,4,5,6,8,9
8	**Angles and their measure** 10.1 Types of Angles 10.2 Conversion from radians to degrees and vice-versa **Online Quiz 27** 10.3 Length of an Arc and area of a Sector **Online Quiz 28** **Worksheet 10 – Moodle Online Test 10**	3	10,11,12,13
	Trigonometry 11.1.1 Definition and Properties of a Right Triangle. 11.1.2 The Pythagoras Theorem **Online Quiz 29**	2	

WEEK No.	TOPICS	HOURS	LEARNING OUTCOMES
9	11.2 Trigonometric ratios in a Right Triangle 11.2.1 The Six Trigonometric Ratios **Online Quiz 30** 11.2.2 Fundamental Trigonometric Identities 11.2.3 Applications of Trigonometric Identities **Online Quiz 31** 11.3 Solutions of Right Triangles and Applications 11.4 Angles of Elevation and Depression **Online Quiz 32** **Worksheet 11- Moodle Online Test 11**	5	11,12,13
10	**Plane Coordinate Geometry** 12.1 The Rectangular Coordinate System 12.2 Distance between two points 12.3 Gradient or Slope of a line **Online Quiz 33** 12.4 Equation of a straight line 12.5 Drawing graph of the straight line function based on its equation 12.6 Parallel and perpendicular lines **Online Quiz 34** **Worksheet 12 – Moodle Online Test 12**	5	14
11	**The Circle & Symmetry** 13.1 Centre and radius of the circle, tangent lines **Online Quiz 35** 13.2 Symmetry of Graphs **Worksheet 13 -Online Test Online Test 13** **Revision of Units 9 – 13**	5	14,15
12	**Final Exam(Topics: Units 9, 10, 11, 12, 13)**		
	TOTAL NUMBER OF TEACHING HOURS	55	

Indicative Reading		
Title/Edition/Author	**Publisher**	**ISBN**
College Algebra with Trigonometry, (9th Edition 2010), Raymond A. Barnett, Michael Ziegler and Karl Byleen, David Sobecki,	McGraw Hill	9780077350109
Basic Engineering Mathematics, (8th Edition 2021), Bird J.	Routledge	9780367643676
Engineering Mathematics, (8th Edition 2020), Stroud K.A and Booth D.J,	Red Globe Press	9781352010275

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Contents

Note: All the content in this workbook will be done without the use of electronic Calculators.

(Unit –1) Basic Theory of Numbers and Operations

1.1. Basic Theory of Numbers

Useful Definitions:

Natural Numbers

They are the numbers you usually use for counting and they will continue on into infinity.

$N = \{ 1, 2, 3, 4, \dots \}$

Whole Numbers

Whole numbers are all natural numbers including zero (0)

$W = \{0, 1, 2, 3, 4, \dots\}$

Integers

Integers include all whole numbers and their negative counterparts.

$\text{Z} = \{0, \pm 1, \pm 2, \pm 3, \dots\}$

Rational Numbers

Definition (1):

All whole numbers, integers, and the numbers in the form of $\frac{p}{q}$ **,**where p and q are integers and $q \neq 0$, are rational numbers.

Examples:

$\frac{3}{5}, \frac{-2}{7}, 4, -5, 0, \sqrt{4}, \sqrt{25} \dots$

Definition (2):

Terminating Decimals are rational numbers.

Examples:

$\frac{3}{2} = 1.5, -2.547,$ and 3.1732

Definition (3):

Non-terminating repeating decimals are rational numbers.

Examples:

$0.3333333333333 \dots$

$10.272727272727 \dots$

Irrational Numbers

Non-terminating and non-repeating decimals are called irrational numbers.

Examples:

$\sqrt{3} = 1.73205\,08075\,68877 \dots$

$4.56247943 \dots$

$\pi = 3.14159\,26535\,89793 \dots$

$e = 2.7182818284590 \dots$

Real Numbers

Both rational and irrational are real numbers.

Even numbers: These are all the integers that are exactly divisible by 2.

Examples: $\{0, \pm 2, \pm 4, \pm 6, \pm 8, \dots\}$

Odd numbers: These are all the integers which are not exactly divisible by 2.

Examples: $\{\pm 1, \pm 3, \pm 5, \pm 9, \dots\}$

Prime numbers: These numbers have exactly two factors, namely: 1 and the number itself.

Examples: $\{2, 3, 5, 7, 11, \dots\}$

Composite numbers: These are numbers which are not prime or which have more than two factors.

Examples: $\{4, 6, 8, 9, 10, \dots\}$

Note: 1 is neither prime nor composite.

Perfect Squares:

A perfect square is an integer which is the square of another integer, that is, n^2.

Note: Since a negative times a negative is positive, a perfect square is always positive.

Examples: $\{1, 4, 9, 16, 25, 36, 49, \dots\}$

Perfect Cubes:

A perfect cube is the result of multiplying a number three times by itself.
Such as: $a \times a \times a = a^3$.
We can also say that perfect cubes are the numbers that have exact cube roots.

Examples:
$\{1, 8, 27, 64, 125, 216, 343, 512, \dots\}$

Square roots:

A square root of a number is a value that can be multiplied by itself to give the original number.

$$3 \xrightarrow{\text{Square}} 9 \qquad 9 \xrightarrow{\text{Square root}} 3$$

3 squared is 9, so a square root of 9 is 3

Examples:

$\sqrt{1} = 1,\ \sqrt{4} = 2, \sqrt{16} = 4, \sqrt{25} = 5,$

$\sqrt{36} = 6$

Cube roots:

The cube root of a number is a special value that when cubed gives the original number.

Examples:

$\sqrt[3]{1} = 1\ ,\ \sqrt[3]{8} = 2\ ,\quad \sqrt[3]{27} = 3,$

$\sqrt[3]{64} = 4\ ,\ \sqrt[3]{125} = 5\ ,\ \sqrt[3]{216} = 6$

Class Activity 1.1

1. Classify all the following numbers as natural, whole, integer, rational, or irrational. Indicate by putting a tick (✓) on all that apply**.**

	Natural	Whole	Integers	Rational	Irrational
117					
0					
-12.6439					
$\frac{-1}{2}$					
6.36					
π					
$0.77777\ \dots$					
$0.7457217\ \dots$					

2. Circle the prime numbers

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40

3. Circle the composite numbers

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40

4. Circle the even numbers

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20

1.2 Arithmetic Operations and Fundamental Laws of Operations

1.2.1 The Four Basic Operations

Sign	Key word	Example
$+$	Add	Add the numbers 2 and 4 $= 2 + 4$
	Addition	Addition of 3 and b means: $3 + b$
	plus	x plus y refers to : $x + y$
	Sum	Sum of 5 and 8 means: $5 + 8 = 13$
	Total	The Total of 2, 1 and 6 means: $2 + 1 + 6 = 9$
	Put together	Put together 1 and 3 means: $1 + 3$
	Increase	Increase 6 by 3 is the same as: $6 + 3$
$-$	Subtract	Subtract 2 from 7 means: $7 - 2$
	Subtraction	Subtraction of 4 from 6 means: $6 - 4$
	minus	12 minus 4 means: $12 - 4$
	difference	The difference between 7 and 5 is $7 - 5 = 2$
	take away	10 take away 3 means : $10 - 3$
$\times$	Multiply	Multiply the numbers 2 and 4 $= 2 \times 4$
	Multiplication	Multiplication of 3 and b means: $3 \times b$
	times	7 times 2 refers to : 7×2
	Product	Product of 5 and 8 is 40 means: $5 \times 8 = 40$
	Of	Half Of 6 means: $½ \times 6 = 3$
	groups of	2 groups of 3 means: 2×3
	lots of	3 lots of 6 books is the same as: $3 \times 6 = 18$ books
$\div$	Divide	Divide 6 by 2 means: $6 \div 2$
	Division	Division of 12 into 3 equal parts means: $12/3 = 4$
/	Share equally	Share 20 Oman rials equally among 5 people means: $20 \div 5 = 4$ Oman rials each.
—	Over	2 over 5 means : $\frac{2}{5}$
Quotient: The answer after we divide one number by another.		dividend ÷ divisor = quotient. e.g.in 12 ÷ 3 = 4, 4 is the quotient.
Remainder: An amount left over after division		$19 \div 5 = 3$ remainder 4

Class Activity 1.2.1

Answer the questions as required.

I. ADDITION

1)Find the sum of 387 and 45.

2) 4906 + 274 + 38 = ___________

II. SUBTRACTION

1) 1 574 – 698 = _____________

2) Find the difference between 9327 and 459.

3) Subtract 385 from 2500.

III. MULTIPLICATION

1) Multiply 25 by 421

2) What is the product of 41 and 20?

3) What is $\frac{2}{3}$ of 60?

IV. DIVISION

1) $72 \div 8 =$______

2) Divide 36 by 9:

3) What is the quotient when 20 ÷ 4 ?

4) What is the remainder when 40 ÷ 3 ?

1.2.2 Fundamental Laws of Operations in Mathematics:

1. **Commutative Laws(rules):**

   i) $a + b = b + a$ (addition)

   ii) $ab = ba$ (multiplication)

   Order doesn't matter

   Examples: i) $4 + 6 = 6 + 4$

   ii) $3 \times 4 = 4 \times 3$

2. **Associative Laws(rules):**

   i) $a + (b + c) = (a + b) + c$ (addition)

   $ii)$ $a(bc) = (ab)c$ (multiplication)

   Examples:

   i) $2 + (3 + 5) = (2 + 3) + 5$

   ii) $4 \times (5 \times 2) = (4 \times 5) \times 2$

3. **Distributive Laws(rules):**

   i) $a(b + c) = ab + ac$

   ii) $(b + c)a = ba + ca$

   Examples:

   i) $4(5 + 3) = 20 + 12 = 32$

   ii) $(2 + 8)5 = 10 + 40 = 50$

Class Activity1.2.2

1. Which of the following shows the Distributive Law/property?

   a) $4 \times (5 \times 2) = (4 \times 5) \times 2$

   b) $4 \times (5 + 2) = 4 \times 5 + 4 \times 2$

   c) $4 + (5 + 2) = (4 + 5) + 2$

2. Name the Law/property used in the following:

   i) $(2 + 1) + 4 = 2 + (1 + 4)$

   ii) $3 + 7 = 7 + 3$

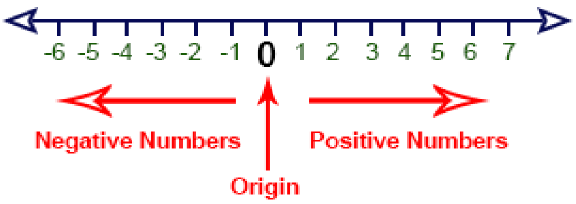
   iii) $3(2 + 5) = 3 \times 2 + 3 \times 5$

   iv) $2 \times 5 = 5 \times 2$

   v) $3 \times (2 \times 5) = (3 \times 2) \times 5$

1.3. Directed numbers and their operations

A ***directed number*** is a number which has a + sign (positive) or a – sign (negative) attached to show its **direction.** Example: -2 and +2



Operations with Directed Numbers:

A number line like the one above can be used to add or subtract directed numbers.

1. Adding & Subtracting numbers

Using a number line perform the following operations**:**

$2 + 2 \quad =$

$-2 - \ 2 =$

$-5 + 3 \ =$

$5 - 3 \quad =$

$-2 + 2 \ =$

2. Multiplication and division of directed numbers

To multiply or divide directed numbers the following rules apply:

$(+) \times (+) = (+)$ Positive multiplied by Positive equal Positive

$(+) \div (+) = (+)$ Positive divided by Positive equal Positive

$(+) \times (-) = (-)$ Positive multiplied by Negative equal Negative

$(+) \div (-) = (-)$ Positive divided by Negative equal Negative

$(-) \times (+) = (-)$ Negative multiplied by Positive equal Negative

$(-) \div (+) = (-)$ Negative divided by Positive equal Negative

$(-) \times (-) = (+)$ Negative multiplied by Negative equal Positive

$(-) \div (-) = (+)$ Negative divided by Negative equal Positive

Multiply the following directed numbers

$(+2)(+5) =$

$+3 \times -4 =$

$(-4)(+5) =$

$(-5)(-3) =$

$(-3)(-2)(+5) =$

$+2)(-4)(-3)(-10) =$

3. Divide the following directed numbers

$\frac{+15}{+3} =$

$\frac{-20}{+2} =$

$\frac{+16}{-8} =$

$\frac{-20}{-4} =$

Class Activity 1.3

Simplify the following:

1) $(-2^3)\,(-5) + (+3)(-4) - (-10)\,(2) =$

2) $(-2)^2\,(5) + (+3)(-4) - (10)\,(-2) =$

3) $(-2)\,(-5) + (+3)(-4) - (10)\,(-2^2) =$

1.4. Sequence of Arithmetic Operations

In the following example which is true?

$2 + 3 \times 4 = 5 \times 4 = 20$

Or

$2 + 3 \times 4 = 2 + 12 = 14$

The correct answer is

$2 + 3 \times 4 = 2 + 12 = 14$, why?

BODMAS is the rule used to avoid CONFUSION when dealing with the arithmetic operations!

BODMAS

B Brackets

O Orders (Powers)

D Division

M Multiplication

A Addition

S Subtraction

Example 1:

Simplify: $(5 \times 3) + 2^3 - 3 \times 4 \div 2$

Solution: Follow the following steps:

B	Brackets	$15 + 2^3 - 3 \times 4 \div 2$
O	Orders (Power)	$15 + 8 - 3 \times 4 \div 2$
D	Division	$15 + 8 - 3 \times 2$
M	Multiplication	$15 + 8 - 6$
A	Addition	$23 - 6$
S	Subtraction	17

Example 2: Simplify: $2 + 3 \times 4$

Solution: Follow the steps as follows:

B There are no brackets so go to next step

O No powers or orders go to next step

D No division go to next step

M $2 + 3 \times 4 = 2 + 12$

A 2+12= 14

S Not required

Class Activity 1.4

Simplify the following using BODMAS rule:

1. $7 \times 5 - 12 \div 4 + 3$

2. $11 \times 2 - 9 \div 3 + 7$

3. $2 + 8 \times (3 + 6)$

4. $10 \times 4 - 2^2 + 2 \times (15 - 9)$

Worksheet 1

For questions 1 to 13 encircle the correct answer:

1. Which number is **irrational**?
a) 7.3
b) $\frac{3}{2}$
c) π

2. Which number is **irrational**?
a) $\sqrt{2}$
b) $\frac{-5}{6}$
c) 0.1111..

3. Which number is **odd**?
a) 2.75
b) 1.73205 087 …
c) √9

4. Which number is **even**?
a) $\sqrt{2}$
b) 46
c) 0.5555..

5. Which number is **prime**?
a) 6
b) 21
c) 2

6. Which of the following shows **distributive property?**
a) $4 + (1 + 2) = (4 + 1) + 2$
b) $5 \times (3 + 7) = (5 \times 3) + (5 \times 7)$
c) $2 \times 3 = 3 \times 2$

7. Which of the following shows **commutative property?**
a) $6 \times (9 \times 5) = (6 \times 9) \times 5$
b) $8 \times (9 + 5) = (8 \times 9) + (8 \times 5)$
c) $3 \times 2 = 2 \times 3$

8. Which of the following shows **associative property?**
a) $7 \times (9 \times 4) = (7 \times 9) \times 4$
b) $5 \times 9 = 9 \times 5$
c) $5 \times (9 + 4) = (5 \times 9) + (5 \times 4)$

9. Which property is used in $8 + (9 + 5) = (8 + 9) + 5$?
a) Commutative
b) Associative
c) Distributive

10. Which property is used in $5 + 9 = 9 + 5$?
a) Commutative
b) Associative
c) Distributive

11. Which property is used in $5 \times (3 - 7) = (5 \times 3) - (5 \times 7)$?
a) Commutative
b) Associative
c) Distributive

12. Simplify : $2 \times 5 - 16 \div 4 + 7 =$
a) 5.5
b) 13
c) -2

13. Simplify :
$5 \times 4 \div 2 - 3^2 + 2 \times (15 - 9)$
a) 13
b) 22
c) 36

14. Classify each of the following numbers as either **natural, rational or irrational**,by ticking(✓) all that apply:

		Natural	Rational	Irrational
i)	0.5			
ii)	5			
iii)	0.666 ...			
iv)	2.645 ...			

15. Showing all steps in your working, simplify:

i) $2[\,15-(\,52\;\div 13\times 2)-(9-6)^2]+8$

Solution:

ii) $\dfrac{4^2-\;(3-5)\times 2}{4^2-6}$

Solution:

(Unit –2) Set Theory

2.1. Sets, Types of Sets, Subsets, Venn Diagrams

A **Set** is a well-defined collection of objects with a common property or characteristic.

Sets are often denoted by capital letters.

Example: The set of natural numbers is denoted as **N** = {1, 2, 3, 4 …}

Null Set: The set containing no elements is called the **empty set** (*null set*) and denoted by { } or ∅.

Each object in a set is called an **element** or a **member** of the set. Element of sets are denoted by lowercase letters.

Belongs (∈): $a \in A$ means *"a belongs to set A"* or "*a is an element of set A"*.

Example: $2 \in \{1, 2, 3, 4\}$

$a \notin A$ *means* " *a* does **not belong to** set A" or it means " *a is **not an element** of set A"*.

Example: $5 \notin \{1, 2, 3, 4\}$

Sets are designated using the following four methods:

1) ***Word description(Sentence):***

Example 1: A is the set of positive even numbers less than 10.

2) ***Listing method:***

Example 2: $A = \{2, 4, 6, 8\}$

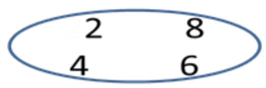
3) ***Set-builder notation method:***

Example 3:

A = {*x* | *x* is an positive even number less than 10} or

$A = \{x : 0 < x < 10, x \text{ is even}\}$

4) Venn diagrams:



Venn Diagram

Class Activity 2.1.1

Write each set using the listing method.

1) {*x* | *x* is a natural number between 3 and 8}

Ans:

2) $\{x \mid -3 < x < 6, x \text{ is an even number}\}$

Ans:

3) {*x* | *x* is a month starting with B}

Ans:

4) {*x* | *x* is a letter in "MATHEMATICS"}

Ans:

Cardinality: The number of elements in a set is called the **cardinal number**, or **cardinality** of the set.

The symbol **n (*A*)**, read "number of elements of ***A***," represents cardinality of set *A*.

Class Activity 2.1.2

Find the cardinal number of each set.

1) *K* = {a, l, g, e, b, r}

Ans :

2) A is the set of positive integers less than 6.

Ans :

3) ∅

Ans :

4) B = {*x* | *x* is a letter in "STATISTICS"}

Ans :

Finite Set: If the **number of elements** in a set is **finite (can be counted or listed definitely)** then the set is called a **finite set**.

Example: The set of odd numbers from 1 to 10: is $\{1,3,5,7,9\}$.

Infinite Set: If the **number of elements** in a set is **not finite (cannot be counted or not listed definitely) then** the set is called an **infinite set**.

Example:

The set of odd numbers $\{1,3,5,7,9\ ...\}$ is an infinite set.

Equality of Sets: Two sets are equal if they contain the same elements in whatever order they are listed.

1) $A = \{9,2,7,-3\}$, and $B = \{7,9,-3,2\}$

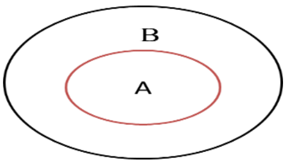
   In this case: $A = B$

2) A = {dog, cat, horse},
   B = {cat, horse, squirrel, dog}

   In this case: $A \neq B$

Subset: $A \subset B$ read as "A is a subset of B".

$A \subset B$ if each element of set A is also an element of set B.



Note

1: $A \subset A$ (Any set is a subset of itself)

2: $\emptyset \subset A$ (Empty set is a subset of any set)

Examples:

1) If $A = \{3,9\}$, $B = \{5,9,1,3\}$, then

$A \subset B$

2) If $A = \{\ \}, B = \{2,3,4\}$, then $A \subset B$

3) $If\ A = \{1,3\}, B = \{3,1\}$, then
$A \subset B$ **or** $A \subseteq B$

4) A = {2, 3, 5}, B = {5, 9, 1, 3}, then

$A \not\subset B$

Note:
$A \subseteq B$: Subset: A has some (or all) elements of B
e.g. If $A = \{2,3\}, B = \{3,2\}$, then $A \subseteq B$

$A \subset B$: Proper Subset: A has some elements of B
e.g If $A = \{2,3\}, B = \{2,3,4,5\}$
then, $A \subset B$

Class Activity 2.1.3

1) State whether the sets in each pair are equal or not.

   i) $A = \{a,b,c,d\}$ and $B = \{a,c,d,b\}$

   ii) $P = \{2,4,6\}$ and
   $Q = \{x \mid x$ is an even natural number less than $10\}$

2) List all the subsets of each of the following sets

a) {1,2}

b) List the elements of the subset of the set

$A = \{-5, 1, \sqrt{2}, 3, \frac{10}{3}, 7\}$ consisting of Natural numbers.

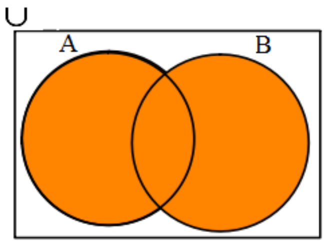
3) If $X = \{1,3,5\}$ and $Y = \{2,3,4,5,6\}$

Is $X \subset Y$?

2.2. Union and intersection of sets

Union* (∪)**: For sets A and B, their ***union $A \cup B$ is the set containing all elements that are either in A, **or** B (**or** in **both**). This can be expressed as $\{x: x \in \mathrm{A}$ **or** $x \in B\}$

The shaded portion in the Venn diagram shows $A \cup B$



Examples:

1) If $A = \{2, 4, 6\}$ and $B = \{3, 5, 7\}$, then find $A \cup B$.

Answer: $A \cup B = \{2, 3, 4, 5, 6, 7\}$

2) If $A = \{-1, 0, 1\}$ and $B = \{0, 1, 2, 3\}$, then find $A \cup B$.

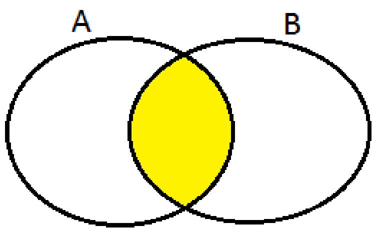
Answer: $A \cup B = \{-1, 0, 1, 2, 3\}$

Note: $A \cup B$ contains all elements of set A and all elements of set B but those which are common or found in both sets are written only once.

***Intersection* (∩)**

For sets A and B, their ***intersection*** $A \cap B$ is the set containing all elements that are common in A and in B or $\{x: x \in \mathrm{A}$ **and** $x \in B\}$

The shaded portion in the Venn diagram shows $A \cap B$



Examples: (i)If $A = \{2, 4, 6\}$ and $B = \{3, 4, 5\}$, find $A \cap B$

Answer: $A \cap B = \{4\}$

(ii)If A= {-1, 0, 1} , B= {0, 1, 2, 3},

then find $\{x: x \in \mathrm{A}$ ***and*** $x \in B\}$

Answer: {0,1}

(iii) $\{a, b, c\} \cap \{2, 3\} = \emptyset$

Class Activity 2.2

1. If $A = \{2, 5, 9\}$, $B = \{1, 3, 4, 6\}$, list elements of

i) $A \cup B =$

ii) $A \cap B =$

2. If $X = \{1, 3, 5\}$, $Y = \{2, 3, 4, 5, 6\}$,

Draw a Venn Diagram to show sets X and Y.

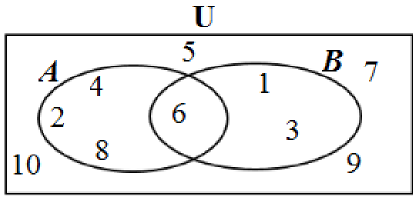
3. If $A = \{3, 6, 9\}$, $B = \{1, 3, 4, 6, 9\}$, draw a Venn Diagram to show sets A and B.

Universal Set (U): This is the Set of all possible value of interest to us.

Example: Universal Set can be $U = \{1,2,3,4,5,6,7,8,9,10\}$, with

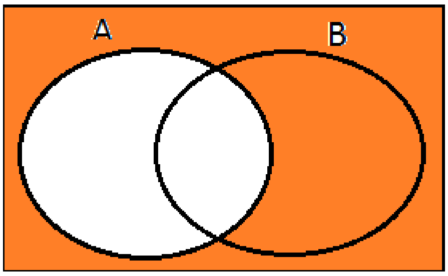
$A = \{2,4,6,8\}$, $B = \{1,3,6\}$.

A Venn diagram showing the relationship between universal set $\boldsymbol{U}$ and sets $\boldsymbol{A}$ and $\boldsymbol{B}$ is shown below:



Complement of a Set

A' or A^c read as "set A complement" or "complement of set A", refers to elements not in set A but part of the universal set ($\boldsymbol{U}$).



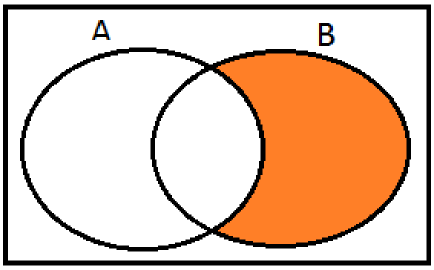
The shaded portion above shows A'

Example:
If $\boldsymbol{U} = \{1,2,3,4,5,6,7,8,9,10\}$, with

$\boldsymbol{A} = \{2,4,6,8\}$, $\boldsymbol{B} = \{1,3,6\}$.

then $\boldsymbol{A}' = \{1,3,5,7,9,10\}$

Set difference: This refers to elements in one set but not in the other.

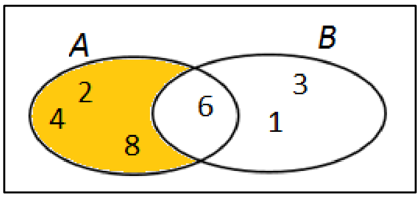


The shaded portion above shows set $B - A$

Example: If $A = \{2,4,6,8\}$, $B = \{1,3,6\}$.

then $A - B = \{2,4,8\}$

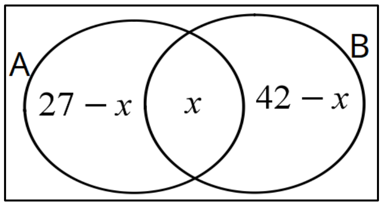
The shaded region in the Venn diagram below shows $A - B$.



2.3 Application of Set theory.

Example 1: In a group of 60 people, 27 like cold drinks and 42 like hot drinks and each person likes at least one of the two drinks. How many like both hot and cold drinks?

Solution:
Let A = Set of people who like cold drinks
B = Set of people who like hot drinks
Let x be the number of people who like both.
Using the Venn diagram below



Therefore $(27 - x)$ like cold drinks only, $(42 - x)$ like hot drinks only.
The total is 60, hence:

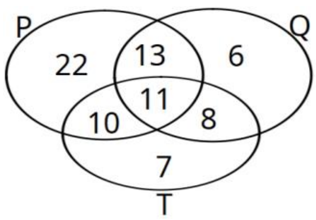
$$(27 - x) + (42 - x) + x = 60$$
$$69 - x = 60$$
$$x = 69 - 60$$
$$x = 9$$

Therefore, 9 people like both hot and cold drinks.

Example 2: Given that $n(P \cup Q \cup T) = 77$, $n(P \cap Q \cap T) = 11$, $n(P \cap Q) = 24$, $n(P \cap T) = 21$, $n(Q \cap T) = 19$, $n(P) = 56$, $n(Q) = 38$, and $n(T) = 36$.
Find $n(P' \cap Q \cap T)$.

Solution: A Venn diagram can help us.

We start in the middle with common elements for all 3 sets:
$n(P \cap Q \cap T) = 11$



We then fill-in the rest of the subdivided intersection spaces by subtracting the 11 from $n(P \cap Q) = 24$, $n(P \cap T) = 21$, and $n(Q \cap T) = 19$.
The number of elements outside the intersections are the last ones to be filled in checking the total number of elements for each set.
$\therefore \boldsymbol{n(P' \cap Q \cap T) = 8}$

Class Activity 2.3

1) If the universal set U, with subsets M and N are described as follows:

$U = \{x: 0 \leq x \leq 15, \quad x \text{ is natural}\}$
$M = \{x | x \text{ is an odd number less than } 15\}$
$N = \{x | x \text{ is a prime number less than } 15\}$

i) List the elements of set U and state n(U)

ii) List the elements of set M.

iii) List the elements of set M'

iv) List the elements of set $M \cup N$.

v) List the elements of set $M \cap N$.

vi) List the elements of set $M - N$.

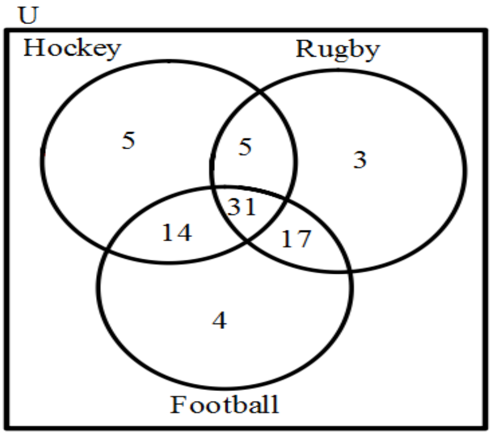
vii) Draw a Venn diagram to show the relationship between sets $U, M \text{ and } N$

2) In a group of 100 persons, 72 people can speak English and 43 can speak Arabic.

i) How many can speak both English and Arabic?

ii) How many can speak English only?

3) Some students were asked which sports they enjoy from Football, Hockey and Rugby. Their responses were represented on a Venn diagram below.



i) How many students enjoy all three sports?

ii) How many students enjoy football and rugby?

iii) How many students were asked altogether?

iv) How many students enjoy hockey and rugby but not football?

Worksheet-2

1. Determine if each statement is **True** or **False.** Underline your answer.

   i) $4 \in \{3,4,6\}$ (True / False)

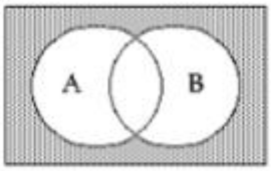
   ii) $3 \notin \{3,4,6\}$ (True / False)

   iii) $\{1,2\} \subset \{1,3,6\}$ (True / False)

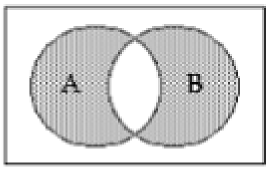
   iv) $\{6,3,5\} = \{3,\ 5,\ 6\}$ (True / False)

2. Which of the Venn diagrams below correctly shows $A' \cap B'$

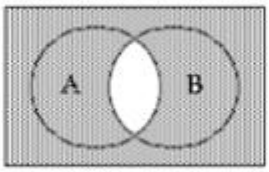
a)



b)



c)



3. Write down all subsets of $A = \{1,2,3\}$

4. Using the listing method, list the elements of the following sets and state their cardinal numbers, n(S).

i) S = {*x / x is an even integer between -5 and 5*}

ii) S ={ *x / x is a month starting with 'J'* }

iii) S = {*x / x is a letter in the word 'COLLEGE'*}

5. If $A = \{1,2,3,5,10\}$ and

$B = \{-3,2,3,\ 5,15,20\}$

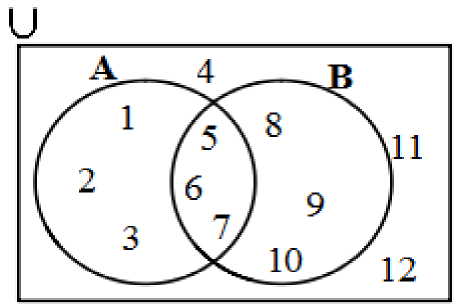
i) Find $A \cup B$

ii) Find $A \cap B$

iii) Is $A \subset B$? (Yes / No)

iv) Draw a Venn diagram to show the relationship between set A and set B.

6. The elements of each set are shown in Venn diagram given below,



State the cardinal number of each set:

(i) $n(A) =$

(ii) $n(U) =$

(iii) $n(B') =$

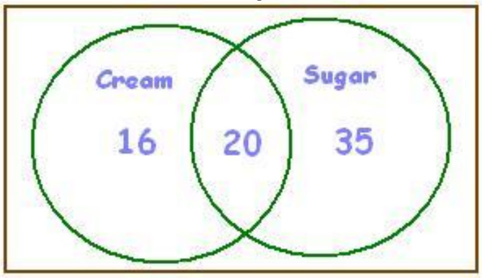
(iv) $n(A \cup B)' =$

(v) $n(A \cap B)' =$

(vi) $n[U - (A \cup B)] =$

7. In a class of 50 students, each of the students passed either in mathematics or in science or in both. 10 students passed in both and 28 passed in science. Find how many students passed in mathematics only?

8. Stephen asked 100 coffee drinkers whether they like cream or sugar in their coffee. According to the Venn diagram below, how many like



i) Cream?

ii) Sugar but not cream?

iii) Cream and sugar?

iv) Cream or sugar?

9. In a town, 800 people are selected by random types of sampling methods. 280 go to work by car only, 220 go to work by bicycle only and 140 use both ways then

i) How many people use at least one of both transportation types?

ii) How many people go by neither car nor bicycle?

10. In a tennis tournament, 100 players took part in the singles only, 31 players took part in the doubles only. The number of players that took part in the singles were equal to twice the number of players who took part in the doubles. How many players took part in both the singles and the doubles?
(*You may use a Venn diagram to help you).*

11. Given, $n(U) = 60$, $n(A) = 34$, $n(B) = 22$, and $n(A \cap B) = 8$.

Find $n(A \cup B)'$. (*You can use a Venn diagram).*

12. A total of 20 trucks were tested at a checkpoint.
6 trucks failed the test for brakes (B)
7 trucks failed the test for lights (L)
9 trucks passed the tests for both brakes and lights.
How many trucks failed both tests for brakes and lights? (*You can use Venn diagrams to find the answer*.)

(Unit-3) Basic Arithmetic

3.1 Factors and Multiples of a number

Factor: A **factor** is a whole number which divides exactly another whole number, leaving no remainder or remainder zero(0).

Example:

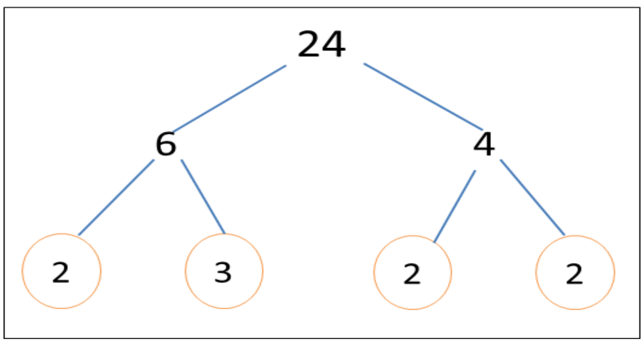
Factors of 24 are $1, 2, 3, 4, 6, 8, 12, 24$

Prime number: A number that has only two (2) factors, namely: 1 and itself.

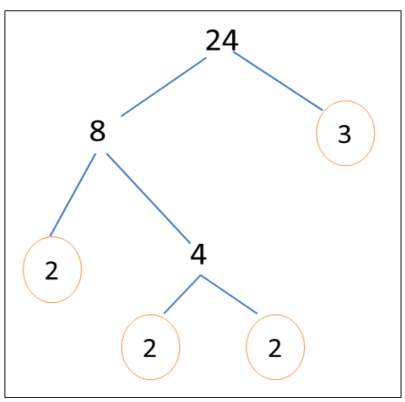
Examples of prime numbers are $2,3,5,7, \ldots$

How do you express or write 24 as a product of its prime factors only?

We use the factor tree method as shown below



OR



$\therefore 24 = 2 \times 2 \times 2 \times 3,$
as a product of its prime factors arranged in ascending order.

Multiples of a number

Multiples of a number are numbers that can be divided into by the number a certain number of times exactly without leaving a remainder.

Examples

1) $3, 6,9,12,15,30, \ldots$ are multiples of 3
2) $5,10,15,20,25, \ldots$ are multiples of 5

Class Activity 3.1

1. Which one is a multiple of 4 but not a factor of 32?

a) 8

b) 16

c) 20

2. Which one is a factor of 48 but not a multiple of 4?

a) 8

b) 6

c) 16

3. Mohammed thinks of a number between 1 and 20 which has exactly 5 factors. What is the number that Mohammed thinks?

a) 10

b) 16

c) 8

4. Write down the following as products of their prime factors using the FACTOR TREE method

i) 56 ii) 105

3.1.1 Highest Common Factor (HCF)

The H.C.F. of a set of numbers is the highest number which is a common factor of each of the numbers.

Example: Find the HCF of 18 and 30.

Factors of 18 are $1, 2, 3, \mathbf{6}, 9, 18$

Factors of 30 are $1, 2, 3, 5, \mathbf{6}, 10, 15, 30$

The highest common factor (HCF) on both lists is 6.

So the HCF of 18 and 30 is 6.

For large or group of numbers we can **use prime factors** (factor trees) **to find HCF**.

Example 1: Find the HCF of $24, 54$ and 42

Step 1: Find their prime factors in index form

$24 = \mathbf{2} \times 2 \times 2 \times \mathbf{3} = \mathbf{2^3} \times 3^1$
$54 = \mathbf{2} \times \mathbf{3} \times 3 \times 3 = 2^1 \times 3^3$
$42 = \mathbf{2} \times \mathbf{3} \times 7 = 2^1 \times 3^1 \times 7^1$

Step 2: Find the product of common factors with lowest powers to get the HCF

It is noted that the factors 2 and 3 are common in all the numbers and their product with lowest powers is: $2^1 \times 3^1 = 6$ is called the Highest Common Factor (HCF) of the numbers $24, 54$ and 42

Example 2: Find the HCF of $36, 60, 108$ and 240

Step1) The prime factors of above numbers are:

$36 = \mathbf{2} \times \mathbf{2} \times \mathbf{3} \times 3 \qquad = 2^2 \times 3^2$

$60 = \mathbf{2} \times \mathbf{2} \times \mathbf{3} \times 5 \qquad = 2^2 \times 3^1 \times 5^1$

$108 = \mathbf{2} \times \mathbf{2} \times \mathbf{3} \times 3 \times 3 \qquad = 2^2 \times 3^3$

$240 = \mathbf{2} \times \mathbf{2} \times \mathbf{3} \times 2 \times 2 \times 5 = 2^4 \times 3^1 \times 5^1$

Step 2) $HCF = 2^2 \times 3^1 = 12$

Check that $36, 60, 108$ and 240 are exactly divisible by 12.

Class Activity 3.1.1

Find the H.C.F of the following set of numbers:

1) 24 *and* 36

2) $10, 40$ *and* 60

3) Find the number which divides 168 and 96 leaving 6 as remainder?

3.1.2 Lowest Common Multiple (LCM)

The **LCM** of a set of numbers is the lowest or smallest number which is a common multiple of each of the numbers.

Example: The lowest common multiple (LCM) of 3 and 4 is the lowest number which is in both multiples of 3 and 4.
Multiples of 3 are: $3, 6, 9, 12, 15, \ldots$
Multiples of 4 are: $4, 8, 12, 16, \ldots$
The first number which comes in both is 12
So 12 is the LCM of 3 and 4.

For large numbers or group of numbers the LCM can be found by using prime factors (factor tree).

Example: Determine the L.C.M of the following set of numbers; $24, 54\ and\ 42.$

Step 1: Express each of them as a product of its prime factors

$24 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3^1$

$54 = 2 \times 3 \times 3 \times 3 = 2^1 \times 3^3$

$42 = 2 \times 3 \times \mathbf{7} \quad = 2^1 \times 3^1 \times 7^1$

Step 2: Find the product of each different prime factor, from all the numbers, with the highest power that occur in any number to get the LCM.

L.C.M = $2^3 \times 3^3 \times 7^1$ **or**

L.C.M = $\mathbf{8 \times 27 \times 7 = 1512}$

<u>Class Activity 3.1.2</u>

1) Find the LCM of 20 and 35.

2) Find the LCM of the following set of numbers: $10, 15$ and 40

3.1.3 Application of LCM and HCF

a) The relationship between LCM and HCF of any two numbers.

For any two numbers:

HCF × LCM = First number × Second number

Example: LCM of 16 and 24 is 48;

HCF of 16 and 24 is 8;

∴ LCM × HCF = 48 × 8 = **384**

Product of the numbers = 16 × 24 = **384**

b) We use HCF method in following ways:

i) To split things into smaller sections.
ii) To equally distribute any number of sets of items into their largest grouping.
iii) To arrange something into rows or groups.

Example (1): A teacher has two classes of 42 and 56 students each. He wants to create small workgroups of equal numbers in each class. If no students should be left over, how many students should be put in each group?
Solution: Since he must consider the numbers 42 and 56 at the same time, HCF will help.

$42 = 2 \times 3 \times 7$
$56 = 2 \times 2 \times 2 \times 7$

HCF$= 2 \times 7 = 14$

Hence in each class, each group should have 14 students.

Example (2): Ahmed has two pieces of ribbon of lengths 18 cm and 24 cm respectively. He wants to cut both pieces into smaller pieces of equal length that are as long as possible. What would be the length of each smaller piece?

Solution: HCF of 18 and 24 is 6. Therefore, the length of each smaller piece is 6cm.

c) We use LCM method in following ways:

i) In an event that is or will be repeating over and over.
ii) To get multiple items in order to have enough.
iii) To analyse when something will happen again at the same time.

Example: Ahmed exercises every 12 days and Said every 18 days. Ahmed and said both exercised today. After how many days they exercise together again?

Solution:
We need to find the LCM of 12 and 18

$12 = 2^2 \times 3^1$
$18 = 2^1 \times 3^2$

Hence LCM $= 2^2 \times 3^2 = 36$

So Ahmed and Said will exercise together again after 36 days.

Class Activity 3.1.3

1) The highest common factor and lowest common multiple of two numbers are 6 and 36 respectively. One number is 12, find the other.

2) A bell rings every 18 seconds, another every 60 seconds. At 5.00 pm the two ring simultaneously. At what time will the bells ring again at the same time?

3) A salesman goes to Salala every 15 days for one day and another every 24 days, also for one day. Today, both are in Salala. After how many days both salesman will be again in Salala on same day?

4) Ibrahim has two metal rods of 45m and 60m respectively. He wants to cut both rods into short lengths of equal measurement. How long should each short piece be?

3.2 Reducing Fractions to Simple form

A **fraction** is written in this form:

$$\frac{a}{b} \rightarrow \frac{\text{Numerator}}{\text{Denominator}}$$

Type of Fractions

- Proper Fraction
  Eg. $\frac{3}{5}$
- Improper or top heavy fraction
  Eg. $\frac{7}{2}$
- Mixed fraction
  Eg. $2\frac{1}{2}$

Conversion of mixed fraction to improper fraction: $a\frac{b}{c} = \frac{c \times a + b}{c}$

Example: $2\frac{1}{3} = \frac{3 \times 2 + 1}{3} = \frac{7}{3}$

Conversion of improper fraction to mixed fraction:

$$\frac{\text{Dividend}}{\text{Divisor}} = \text{Quotient}\frac{\text{Remainder}}{\text{Divisor}}$$

Example: $\frac{7}{2} = 3\frac{1}{2}$

Equivalent Fractions: $\frac{a}{b} = \frac{ka}{kb}$

Example: $\frac{1}{2} = \frac{5 \times 1}{5 \times 2} = \frac{5}{10}$

Reducing a fraction to simplest form:

Do factorization in both the numerator and the denominator, then cancel out common factors.

Example 1: $\frac{15}{35} = \frac{3 \times \cancel{5}}{7 \times \cancel{5}} = \frac{3}{7}$

Example 2: $\frac{24}{36} = \frac{\cancel{2} \times \cancel{2} \times 2 \times \cancel{3}}{\cancel{2} \times \cancel{2} \times 3 \times \cancel{3}} = \frac{2}{3}$

Note: A fraction is in simplest form if its numerator and denominator have no common factors.

Class Activity 3.2

1. Convert the mixed fractions to improper form:

i) $1\frac{1}{2} =$

ii) $3\frac{1}{3} =$

iii) $5\frac{2}{5} =$

2. Convert the improper fractions to mixed fractions:

i) $\frac{7}{2} =$

ii) $\frac{30}{7} =$

3. Reduce the following fractions to their simplest form

i) $\frac{4}{12} =$

ii) $\frac{6}{39} =$

4. Write two equivalent fractions of the following:

i) $\frac{2}{5} =$

ii) $\frac{3}{7} =$

3.3 Addition and Subtraction of fractions

Rules and Steps for adding or subtracting fractions

Rule 1: If the fractions are mixed convert them to improper fractions.

Rule 2: If all the fractions have the same denominators (like/similar fractions), then just add or subtract the numerators and copy the common denominator.

Rule 3: If the denominators of the fractions are different, do the following steps:

Step 1: Find the Least Common Denominator (LCD) which is the Least Common Multiple (LCM) of all the denominators.

Step 2: Using the LCD, convert the fractions to become similar fractions.

Step 3: Combine the fractions into one fraction.

Step 4: Express the fraction in simplest form.

Examples: Simplify the following:

1) $\frac{2}{5}+\frac{1}{5}$

Solution:

Since denominators are the same, add numerators $\frac{2+1}{5}=\frac{3}{5}$

2) $\frac{5}{7}-\frac{2}{7}$

Solution:

$$\frac{5-2}{7}=\frac{3}{7}$$

3) $\frac{5}{7}-\frac{1}{5}$

Solution:

LCM of denominators 7 and 5 is **35**

$$\frac{5}{7}-\frac{1}{5}=\frac{5\times5}{5\times7}-\frac{7\times1}{7\times5}$$

$$=\frac{5\times5-7\times1}{35}$$

$$=\frac{25-7}{35}$$

$$=\frac{18}{35}$$

4) $\frac{2}{3}+\frac{1}{4}-\frac{1}{2}$

Solution:

LCM of denominators 3, 4 and 2 is **12**

$$\frac{2}{3}+\frac{1}{4}-\frac{1}{2}=\frac{4\times2+3\times1-6\times1}{12}$$

$$=\frac{8+3-6}{12}$$

$$=\frac{5}{12}$$

5) $1\frac{2}{3}+2\frac{1}{2}$

Solution:

$$1\frac{2}{3}+2\frac{1}{2}=\frac{3\times1+2}{3}+\frac{2\times2+1}{2}$$

$$=\frac{5}{3}+\frac{5}{2}$$

LCM of denominators 2 and 3 is **6**

$$\frac{5}{3}+\frac{5}{2}=\frac{2\times5+3\times5}{6}$$

$$=\frac{25}{6} \text{ or } 4\frac{1}{6}$$

Class Activity 3.3

1. Simplify: $\frac{1}{2} + \frac{5}{8}$

2. Simplify: $\frac{7}{10} - \frac{2}{5}$

3. Simplify: $\frac{5}{3} + \frac{2}{5} - \frac{1}{2}$

4. Simplify: $\frac{1}{4} - \frac{3}{8} + 1\frac{1}{2}$

5. An electrician has three and seven-sixteenths cm of wire. He needs only two and five-eighths cm of wire for a job. How much wire is remaining?

3.4 Multiplication and Division of Fractions:

Multiply the numerators on their own and multiply the denominators on their own and then simplify.

Note: The cancellation method may also be used.

Example 1: Simplify $\frac{5}{8}\times\frac{3}{7}$

Solution: $\frac{5}{8}\times\frac{3}{7}=\frac{5\times3}{8\times7}=\frac{15}{56}$

Example 2) Simplify $\frac{2}{3}\times1\frac{7}{8}$

Solution: $\frac{2}{3}\times1\frac{7}{8}=\frac{2}{3}\times\frac{15}{8}=\frac{30}{24}=\frac{5}{4}=1\frac{1}{4}$

A shorter way would have involved cancellation method as shown below:

$\frac{\cancel{2}^{1}}{\cancel{3}_{1}}\times\frac{\cancel{15}^{5}}{\cancel{8}_{4}}=\frac{1\times5}{1\times4}=\frac{5}{4}=1\frac{1}{4}$

Division:

Given that a and b are integers different from zero, then $a \div b = \frac{a}{1}\times\frac{1}{b}$

In general $\frac{a}{b}\div\frac{c}{d}=\frac{a}{b}\times\frac{d}{c}$

where $\frac{d}{c}$ **is called the reciprocal of** $\frac{c}{d}$

Example 1: $6\div2=\frac{6}{2}=\frac{6}{1}\times\frac{1}{2}$

Example 2: Simplify $\frac{3}{5}\div\frac{2}{7}$

Solution:

$\frac{3}{5}\div\frac{2}{7}=\frac{3}{5}\times\frac{7}{2}=\frac{21}{10}=2\frac{1}{10}$

Class Activity 3.4

Simplify the following.

1) $\frac{2}{3}\times\frac{6}{5}$

2) $1\frac{2}{5}\times3\frac{1}{2}$

3) $1\frac{2}{9}\times1\frac{4}{5}-\frac{6}{5}$

4) $\frac{1}{2}+2\frac{1}{4}\times3\frac{1}{3}$

5) $\frac{4}{5}\div1\frac{1}{3}$

6) $1\frac{3}{7}\div5$

7) $\frac{1}{2}+26\div3\frac{1}{4}$

8) Osama put in his car tank $\frac{5}{8}$ of a 24 litre container of fuel. Khalil put $\frac{1}{3}$ as much fuel as Osama did. How many litres of fuel did Khalil use?

3.5 Estimation/Rounding off

3.5.1 Estimation/Rounding off (Decimal places)

Rule 1: If the digit to be dropped is 5 or more, then the digit at its left is increased by 1.

Rule 2: If the digit to be dropped is less than 5, then discard the digits on the right and retain the digits on the left without changing the last digit.

Example 1: Round off: 85.7684
correct to i) 1 decimal place.
ii) 3 decimal places.

Solution:
i) $85.7684 =$ 85.8, to 1 decimal place.
(***Rule 1*** *applies*)

ii) $85.7684 =$ 85.768, to 3 decimal places.
(***Rule 2*** *applies*)

Example 2: Round off: 0.007362
correct to i) 2 decimals.
ii) 3 decimals

Solution:
i) $0.007362 = 0.01$ correct to 2 decimals.

ii) $0.007362 = 0.007$ correct to 3 decimals.

Class Activity 3.5.1

1. When $\mathbf{53.6492}$ is rounded off correct to 3 decimal places, the answer is …….
   a) 53.6
   b) 53.649
   c) 53.650

2. When $\mathbf{0.547}$ is rounded off correct to 2 decimal places, the answer is …….
   a) 0.55
   b) 0.54
   c) 0.50

3. When $\mathbf{72.95}$ is rounded off correct to 1 decimal place, the answer is …….
   a) 72.9
   b) 80.0
   c) 73.0

4. Round off $\mathbf{0.6791}$ to 2 decimal places.

5. Round off $\mathbf{23.521}$ to 1 decimal place.

6. Round off $\mathbf{64.796}$ to 2 decimal places.

3.5.2. Estimation/Rounding off (Significant Figures or Digits)

Instead of using the number of **decimal places** to express the accuracy of an answer, **significant figures** can be used.

The number 39.38 is written in 2 decimal places or in 4 significant figures since the number contains four figures.

The number 39.38 is correct to 2 decimal places but it is also correct to 4 significant figures since the number contains four figures.

The rules regarding significant figures are as follows:

Rule 1: Non-zero digits are always significant.

Examples:

i) 25 has **two** significant figures.

ii) 25**000** has **two** significant figures (*zeros in bold are not considered*)

iii) 25.58 has **four** significant figures.

iv) 8.1925 = 8.193 correct to **4** significant figures. (Here last digit is increased by 1)

v) 8.1925 = 8.19 correct to **3** significant figures.

Rule 2: Any zeros between two non-zero digits are significant.

Examples:

i) 506 has **3** significant figures.

ii) 12007 has **5** significant figures.

iii) 20.07 has **4** significant figures.
(*Though **2** decimal places*)

iv) $60\ 256 = 60\ 000$ to 2 significant figures.
Note: Here 3 zeros (0) are added after the first two digits to maintain the number in thousands similar to the original number.

Rule 3: A final zero or trailing zeros in the decimal portion <u>ONLY</u> are significant.

Examples:

i) **0.000**70 has 2 significant figures (zeros in bold are not considered)
(*Though 5 decimal places*)

ii) **0.0**3040 has 4 significant figures (zeros in bold are not considered)
(*Though 5 decimal places*)

iii) 1.0050 has 5 significant figures.
(*Though 4 decimal places*)

Note: The same rounding cut off rules used in decimal rounding also apply to significant figure rounding off, that is:

If the figure (digit) to be discarded is 5 *or more, then the previous figure is increased by* 1, *otherwise the last digit is not increased.*

Examples:

1) $0.40561 = 0.406$ if rounded off to 3 significant figures.

2) $7.236 = 7.2$ if written in 2 significant figures.

Class Activity 3.5.2

Put a circle around the correct answer for questions 1 to 5, **indicate** how many significant figures there are in each of the numbers.

1. 0.00340
  a) 3
  b) 5
  c) 2

2. 14.600
  a) 2
  b) 3
  c) 5

3. 700000
  a) 1
  b) 6
  c) 4

4. 350.670
  a) 3
  b) 4
  c) 6

5. When $\mathbf{42.456}$ is rounded off correct to 2 significant figures, the answer is …
  a) 42
  b) 43
  c) 42.46

6. When $\mathbf{5621}$ is rounded off correct to 3 significant figures, the answer is …
  a) 5600
  b) 5620
  c) 5630

7. Round off $\mathbf{25.271}$ to 3-significant figures.

3.6 Scientific Notation

Standard form of a scientific number

The expression $a \times 10^n$ is a scientific notation, where a is a number between 1 and 10 and n is an integer.

Example1: Distance from earth to sun $= 93\ 000\ 000$ miles $= 9.3 \times 10^7$ miles

Example2: Diameter of atom of gold $= 0.000000342$ m $= 3.42 \times 10^{-7}$ m

3.6.1 Conversion of conventional number to scientific number

Rule 1: When the decimal point is moved 'n' places to the left then power of 10 is positive 'n'.

Example 3: Express these numbers in scientific notation

i) $33\ 600\ 000$

Ans) 3.36×10^7

ii) 502.15

Ans) 5.0215×10^2

Rule 2: When the decimal point is moved 'n' places to the right then power of 10 is negative 'n'.

Example 4: Express these numbers in scientific notation.

i) 0.0045 — **Ans)** 4.5×10^{-3}

ii) 0.000506 — **Ans)** 5.06×10^{-4}

Class Activity 3.6.1

Express these numbers in scientific notation

1) 2056

2) 0.00000377

3) 499800000000

4) 0.034

3.6.2 Conversion of Scientific numbers to conventional (ordinary) numbers

Rule 1: If the power of 10 is positive '$\boldsymbol{n}$' then move the decimal point '$\boldsymbol{n}$' places to the right (fill the vacant places with zero).

Example 5: Express these numbers given in scientific notation in conventional form.

i) 5.0215×10^2

Ans) 502.15

ii) 3.36×10^6

Ans) $3,360,000$

Rule 2: If the power of 10 is negative 'n' then move the decimal point 'n' places to the left (fill the vacant places with zero).

Example 6: Express each number given in scientific notation in conventional form

i) 4.5×10^{-3}

Ans) 0.0045

ii) 5.06×10^{-4}

Ans) 0.000506

Class Activity 3.6.2

Express these numbers in conventional form

1) 4.36×10^6

2) 5.3×10^{-4}

3) 9.66×10^{-5}

4) 8.50×10^2

Worksheet-3

1. Azan pays OMR 12 for 2 magazines. Given that the cost of each magazine is a multiple of 4. What are the possible prices for each of the magazines?

a) OMR 6 and OMR 2

b) OMR 8 and OMR 4

c) OMR 3 and OMR 4

2. The lowest common multiple of 6 and 4 is smaller than the highest common factor of 30 and 45?

a) False

b) True

c) None of the above

3. By first expressing each of the following numbers as products of their prime factors $42, 90$ and 108;
Find the
i) HCF

ii) LCM

4. The HCF of two numbers is 3 and their LCM is 54. If one of the numbers is 27, find the other number.

5. Find the greatest number which divides 305 to leave a remainder of 5 and which also divides 299 to leave a remainder of 5?

6. Find the least length of a rope which can be cut into whole number of pieces of lengths 45 cm, 75 cm and 81 cm?

7. A cinema runs its movies in two different halls 24/7. One movie runs for 90 minutes and the second one runs for 60 minutes. Both movies start at 1.00 p.m. When will the movies begin again at the same time?

8. Simplify $\frac{3}{8} - \frac{1}{2} + \frac{3}{4}$

9. Simplify $2\frac{1}{3} \div 1\frac{1}{4} \times 2\frac{3}{4}$

10. Simplify $10 \times \frac{3}{5}$

11. Simplify $2\frac{3}{11} \div \frac{1}{11}$

12. Simplify $\left(6 - \frac{3}{2}\right) \times \frac{2}{9}$

13. Simplify $\frac{2}{3} \times \left(5 \div \frac{1}{6}\right)$

14. Salim had a box of chocolates, of which he gave 1/2 to his friend Abdullah. Abdullah gave 3/4 of his share to his friend Khalid. What fractional part of the original box of chocolates did Khalid get?

15. A painter had a trough with 22 litres of paint. If each bucket holds $2\frac{3}{4}$ litres, then how many buckets of paint can be poured from the trough?

16. Indicate how many significant figures there are in each of the following numbers.

(i) 246.32

a) 3
b) 5
c) 2

(ii) 100.3

a) 1
b) 3
c) 4

(iii) 0.678

a) 4
b) 3
c) 1

17. Round-off the following numbers:

i) 1.2562 (to 2-significant figures) $=$

ii) 0.1125 (to 2-significant figures) $=$

iii) 50257 (to 2-significant figures) $=$

18. Round-off the following numbers:

i) 49.3915 (to 2-decimal places) $=$

ii) 0.006365 (to 2-decimal places) $=$

19. The number 0.0000413 can be written as

a) 413×10^{-7}
b) 0.413×10^{-7}
c) 4.13×10^{-7}

20. Express in scientific notation 49900000000

21. Express these numbers in conventional form.

i) $5.73 \times 10^{6} =$

ii) $8.66 \times 10^{-5} =$

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