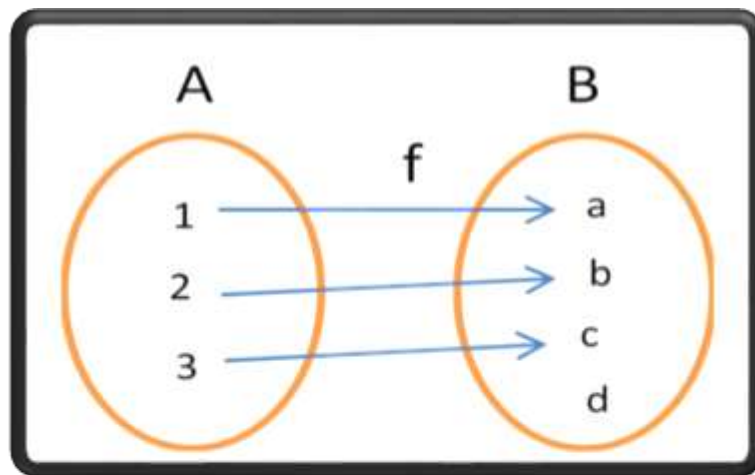
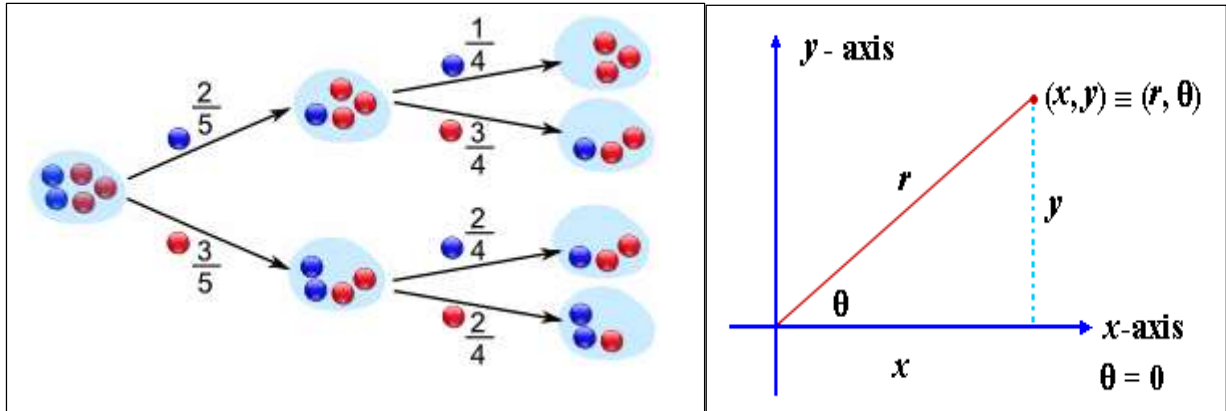




# MILITARY TECHNOLOGICAL COLLEGE



## GSD- Pure Mathematics

MODULE CODE: MTCG1018

# WORKBOOK-1

<b>Learning Outcomes (10 max.) – On successful completion of this module, students should be able to:</b>	
1	Demonstrate understanding of the definition of a function and its graph.
2	Logarithm. Define and manipulate exponential and logarithmic functions and solve problems arising from real life applications.
3	Understand the inverse relationship between exponents and logarithms functions and use this relationship to solve related problems.
4	Understand basic concepts of descriptive statistics, mean, median, mode and summarize data into tables and simple graphs (bar charts, histogram, and pie chart).
5	Understand basic probability concepts and compute the probability of simple events using tree diagrams and formulas for permutations and combinations.
6	Understand the concept and determine average and average speed.
7	Determine the surface areas, the volumes and capacities of common shapes and 3-dimensions figures (square, rectangle, parallelogram, trapezium, cuboid, cone, pyramid and prisms).
8	Represent points given in polar form on a plane and discuss their characteristics. Convert coordinates of a point from Cartesian form to polar form and vice-versa.
9	Carry out inter conversions between decimal, binary, octal & hexadecimal systems and addition and subtraction of binary numbers.
10	Use the law of sines and cosines to solve a triangle and real life problems.

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**Assessment Plan** (Passing Mark: 50 %)

<b>Assessment</b>	<b>Weightage</b>
Continuous Assessment-1	20%
Continuous Assessment-2	30%
Final Exam	50%
Total	100%

**Note:** Only Non-Programmable calculators, prescribed in MTC exam rules, are allowed.

**Attendance Policy:**

<b>Warning</b>	<b>Absence</b>
First	10%
Second	15%
Third	20%

## (UNIT- 1) AVERAGES

### 1.1 AVERAGE

The average of a group of numbers of the same kind is a single number which represent the group.

To find the average of a set of quantities, add the quantities together and divide by the number of quantities in the set. Thus,

$$\text{Average} = \frac{\text{sum of the quantities}}{\text{number of quantities}}$$

#### **Examples:**

1) The following are the marks of 8 students in a class: 8, 20, 3, 8, 15, 9, 17 and 12. What is the average mark of the students?

**Solution:**

$$\begin{aligned}\text{Average mark} &= \frac{8+20+3+8+15+9+17+12}{8} \\ &= \frac{92}{8} = 11.5\end{aligned}$$

2) Find the average age of a team of boys given that four of them are 15 years old each and the other three boys are 14 years old each.

**Solution:**

$$\begin{aligned}\text{Total age of 4 boys at 15 years} &= 4 \times 15 \\ &= 60 \text{ years}\end{aligned}$$

$$\begin{aligned}\text{Total age of 3 boys at 14 years} &= 3 \times 14 \\ &= 42 \text{ years}\end{aligned}$$

$$\text{Total age of all 7 boys} = 60+42 = 102 \text{ years}$$

$$\text{Average age} = \frac{102}{7} = 14.57 \text{ years}$$

3) The average mark of 24 candidates taking an examination is 42. Find what the average mark would have been if one candidate who scored 88 was absent.

**Solution:**

$$\text{Total marks of 24 students} = (24)(42) = 1008$$

$$\text{Total marks of 23 students if a student who scored 88 marks is absent} = 1008 - 88 = 920$$

$$\therefore \text{Average marks of 23 students} = \frac{920}{23} = 40$$

#### **Class Activity**

1) Nasser scored 76, 65, 82, 67 and 85 marks in English, Mathematics, Physics, Chemistry and Computing respectively. What is his average mark?

**Solution:**

$$\text{Average mark} = \frac{76+65+82+67+85}{5} = \frac{375}{5} = 75$$

2) Find the average of the following:

22.2 cm, 12.4 cm, 17.6 cm, and 19.8 cm.

**Solution:**

$$\text{Average} = \frac{22.2+12.4+17.6+19.8}{4} = \frac{72}{4} = 18$$

3) Find the average of the following lengths:

5 m, 11 m, 16 m, 520 cm, 280 cm.

**Solution:**

$$\text{Average} = \frac{5+11+16+5.2+2.8}{5} = \frac{39}{5} = 7.8$$

$$\text{Average} = \frac{5+11+16+5.2+2.8}{5} = \frac{39}{5} = 7.8$$

4) What is the average of the first 5 even natural numbers?

**Solution:**

$$\text{Average} = \frac{2+4+6+8+10}{5} = \frac{30}{5} = 6$$

5) The average of three numbers is 58. The average of two of them is 49. Find the third number.

**Solution:**

6) The average of five numbers is 26.8. The average of the first two numbers is 20 and the average of the last two numbers is 35. What is the middle number?

**Solution:**

7) In a class of 20 students, the average age of 12 boys is 13 years and the average age of 8 girls is 12 years. What is the average age of the whole class?

**Solution:**

8) Find the average mass of 20 boxes if 9 boxes each have a mass of 11 kg, 7 boxes each have a mass of 15 kg and 4 boxes each have a mass of 9 kg.

**Solution:**



## 1.2 AVERAGE SPEED

$$\text{Average speed} = \frac{\text{Total distance travelled}}{\text{Total time taken}}$$

The unit of speed depends on the unit of distance and the unit of time. For instance, if the distance travelled is in kilometres (*km*) and the time taken is in hours (*hr*) then the speed will be stated in kilometres per hour (*km/h*). If the distance is given in metre (*m*) and the time in seconds (*s*) then the speed is in metres per second (*m/s*).

### Examples:

1) A car travels a total distance of 480 *km* in 4 *hours*. What is its average speed?

**Solution:**

$$\begin{aligned}\text{Average speed} &= \frac{\text{Total distance travelled}}{\text{Total time taken}} \\ &= \frac{480}{4} = 120 \text{ km/hr}\end{aligned}$$

2) A car travels a total distance of 48 *miles* in 45 *minutes*. What is its average speed?

**Solution:**  $time = \frac{45}{60} = 0.75 \text{ hours}$

$$\begin{aligned}\text{Average speed} &= \frac{\text{Total distance travelled}}{\text{Total time taken}} \\ &= \frac{48}{0.75} = 64 \text{ miles/hr}\end{aligned}$$

3) An aircraft travels 1440 *nautical miles* in 2 *hours 15 minutes*. What is the average speed?

**Solution:**  $time = 2.25 \text{ hours}$

$$\begin{aligned}\text{Average speed} &= \frac{\text{Total distance travelled}}{\text{Total time taken}} \\ &= \frac{1440}{2.25} \\ &= 640 \text{ nautical miles/hr}\end{aligned}$$

4) A car travels 300 *km* at 120 *km/h* and 120 *km* at 80 *km/h*. Find its average speed.

**Solution:**

$$time = \frac{\text{distance}}{\text{speed}}$$

$$\text{Case(i): } time = \frac{300}{120} = 2.5 \text{ hours}$$

$$\text{Case(ii): } time = \frac{120}{80} = 1.5 \text{ hours}$$

$$\text{Average speed} = \frac{\text{Total distance travelled}}{\text{Total time taken}}$$

$$\begin{aligned}&= \frac{300+120}{2.5+1.5} \\ &= \frac{420}{4} = 105 \text{ km/hr}\end{aligned}$$

### Class Activity

1) A train travels 300 *km* in 4 *hours*. What is its average speed?

**Solution:**

2) A car travels 200 *km* at an average speed of 50 *km/h*. How long does it take?

**Solution:**

3) A bus travels a total distance of 105 *miles* in 90 *minutes*. What is its average speed?

**Solution:**

4) An aircraft travels 3600 *nautical miles* in 4 *hours 30 minutes*. What is the average speed?

**Solution:**

5) A car travels 450 *km* at 100 *km/h* and 90 *km* at 60 *km/h*. Find its average speed.

**Solution:**

6) A car travels 150 *km* at an average speed of 120 *km/h*. On the return journey the average speed is 75 *km/h*. Calculate the average speed over the whole journey.

**Solution:**





## WORKSHEET 1

### Section-A

**Circle the correct answer in the following questions.**

1) In the set  $\{11, 12, 13, 14, 15, 16, 17, 18, 19\}$  , what is the average of the prime numbers?

- (a) 15
- (b) 14
- (c) 8

2) If average of 3 numbers is 7 then sum of the numbers is .....

- (a) 10
- (b) 21
- (c) 4

3) Ahmed scored 42, 34, 37 and 39 marks in English, Math, Physics and Computing. What is his average marks?

- (a) 50.7
- (b) 38
- (c) 28.3

4) The average of 1 m, 60 cm, 20 cm is .....

- (a) 60 cm
- (b) 180 cm
- (c) 6 m

5) The average of 1 hour 25 minutes, 2 hours 40 minutes, 3 hours 35 minutes and 4 hours is .....

- (a) 2 hours 55 minutes
- (b) 2 hours 40 minutes
- (c) 3 hours 15 minutes

6) 105 Minutes = ..... Hours

- (a) 1.45
- (b) 10.5
- (c) 1.75

7) A car travels a distance of 425 km in 4 hours 15 minutes. What is its average speed?

- (a) 100 km/hour
- (b) 102 km/hour
- (c) 1806 km/hour

8) A whale swims at a constant speed of 8 m/s for 16 s. What distance does it travel?

- (a) 128 m
- (b) 2 m
- (c) 0.5 m

9) Ahmed swims at a constant speed of 8 m/s and travels 128 m. How long he swims?

- (a) 1024 seconds
- (b) 16 seconds
- (c) 0.06 seconds

### Section-B

**Show your solution step by step in the following questions.**

1) Omar loves to go bird watching. First day, he saw 7 birds. The second day, he saw 11 birds and the third day, he saw no birds. What is the average number of birds Omar saw each day?

**Solution:**

2) The average of five numbers is 40. The average of the first two numbers is 25 and the average of the last two numbers is 60. What is the middle number?

**Solution:**

3) Find the average mass of 10 boxes if 5 boxes each have a mass of 12 kg, 3 boxes each have a mass of 8 kg and 2 boxes each have a mass of 6 kg.

**Solution:**

4) The average of 26 students taking an examination is 44. Find what the average mark would have been if one candidate, who scored 94 was absent.

5) The average of 20 numbers is 65. While calculating average, the number 42 was incorrectly written as 24. Find the correct average.

**Solution:**

6) A train runs for 3 *hours* at a speed of 55 *km/hr* and for the next 2 *hours* at a speed of 65 *km/hr*. Find the average speed of the train for the 5 *hours* journey.

**Solution:**

7) A car travels 400 *km* at 80 *km/hr* and 360 *km* at 120 *km/hr*. Find its average speed.

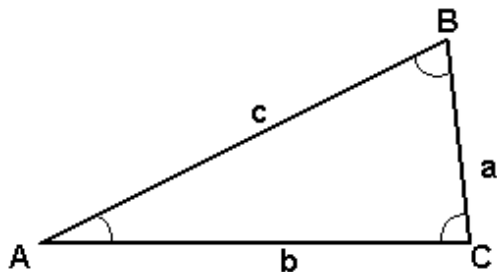
**Solution:**

8) Nasser travels in an airplane a distance of 900 *km*. For three-fourth of the distance, the airplane flies at a speed of 150 *km/hr* and for the rest of the distance, it flies at a speed of 100 *km/hr*. How long does the whole trip take?

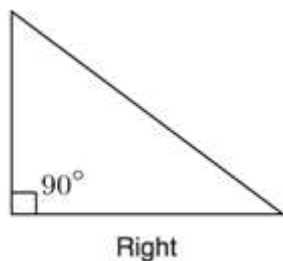
**Solution:**

## (UNIT-2) LAW OF SINES AND COSINES

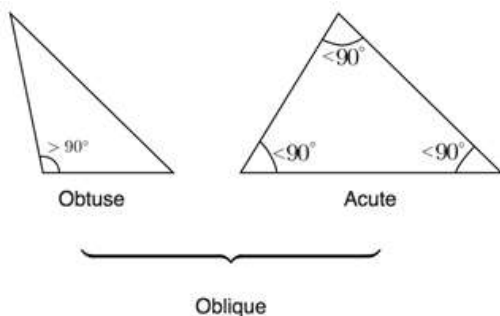
In a triangle, there are always three vertices, three angles and three sides.



A **right triangle** is a triangle in which one angle is  $90^\circ$ .

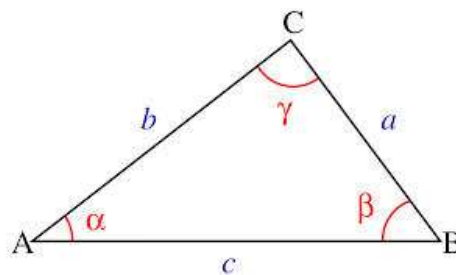


An **oblique triangle** is a **triangle** with no right angle. An **oblique triangle** has either three acute angles, or one obtuse angle and two acute angles.

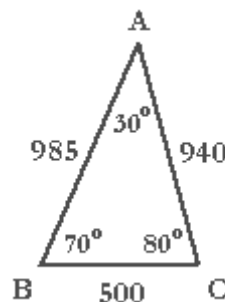


In any **triangle**, the sum of all three angles is equal to 180 degrees.

### Notations used in solving triangle



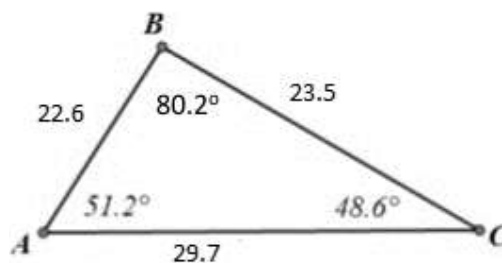
For example in the following triangle:  
 $a = 500$  ;  $b = 940$  ;  $c = 985$  ;  
 $\alpha = 30^\circ$  ;  $\beta = 70^\circ$  and  $\gamma = 80^\circ$



Note: Authors also use A, B, and C for  $\alpha$ ,  $\beta$  and  $\gamma$  respectively.

### Class Activity 1

Find a, b, c,  $\alpha$ ,  $\beta$  and  $\gamma$  in the following triangle.

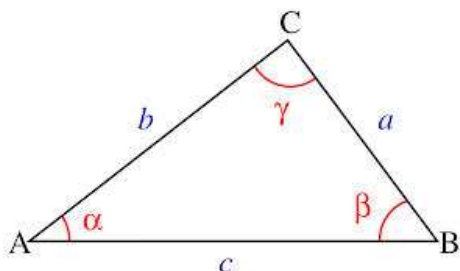


**Answer:**

$a =$  ;  $b =$  ;  $c =$  ;  
 $\alpha =$  ;  $\beta =$  and  $\gamma =$

The law of sines and law of cosines play an important role in solving oblique triangles.

## 2.1 LAW OF SINES



Any side of a triangle is proportional to the sine function of its opposite angle. As per the sine law:

$$\frac{\sin \alpha}{a} = \frac{\sin \beta}{b} = \frac{\sin \gamma}{c}$$

Or 
$$\frac{a}{\sin \alpha} = \frac{b}{\sin \beta} = \frac{c}{\sin \gamma}$$

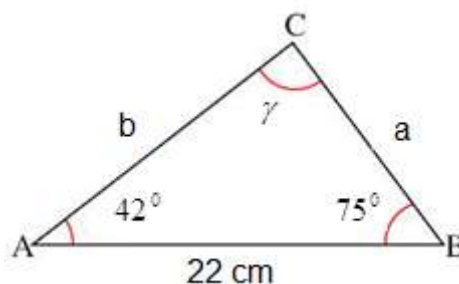
The law of sines is used to solve triangles in the following cases:

- (i) Two angles and any side (ASA or AAS)
- (ii) Two sides and an angle opposite one of them (SSA)

**Note:** For SSA there are ambiguous cases, in which triangle does not exist and will not be discussed in this section.



**Example 1:** Solve the triangle:



(Round the answers up to 2 decimal places)

**Solution:** We are given two angles and the included side, which is the ASA case. Here

$$\alpha = 42^\circ, \beta = 75^\circ \text{ and } c = 22 \text{ cm}$$

**Step (1) Find the third angle**

$$\alpha + \beta + \gamma = 180^\circ$$

$$42 + 75 + \gamma = 180^\circ$$

$$\gamma = 63^\circ$$

**Step (2) Find any of the remaining two sides with the Sine rule**

$$\frac{a}{\sin \alpha} = \frac{b}{\sin \beta} = \frac{c}{\sin \gamma}$$

$$\frac{a}{\sin 42} = \frac{b}{\sin 75} = \frac{22}{\sin 63} \text{----- (1)}$$

From first and third fraction of (1)

$$\frac{a}{\sin 42} = \frac{22}{\sin 63}$$

$$a = \frac{22 \sin 42}{\sin 63} = 16.52 \text{ cm}$$

(Rounded to 2 decimal places)

**Step (3) Find the third side with the Sine rule**

From second and third fraction of (1)

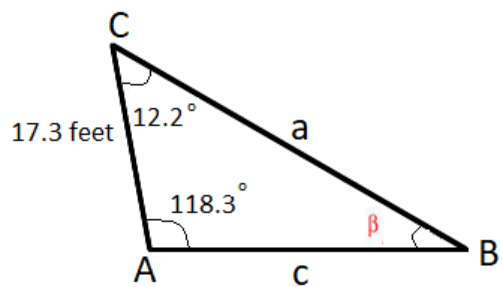
$$b = \frac{22 \sin 75}{\sin 63} = 23.85 \text{ cm}$$

(Rounded to 2 decimal places)

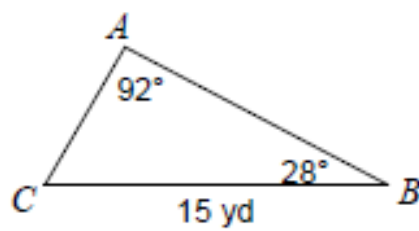
**Class Activity 2**

**(Round the answers up to 2 decimal places)**

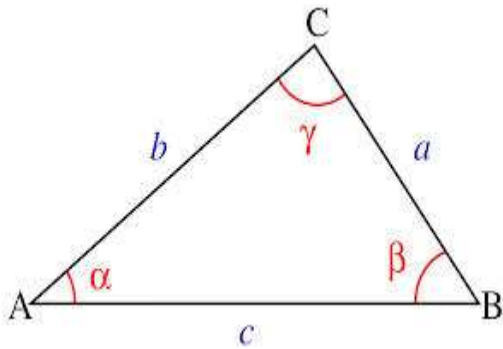
1) Solve the triangle:



2) Solve the triangle:



## 2.2 LAW OF COSINES:



$$1) a^2 = b^2 + c^2 - 2bc \cos \alpha$$

$$2) b^2 = a^2 + c^2 - 2ac \cos \beta$$

$$3) c^2 = a^2 + b^2 - 2ab \cos \gamma$$

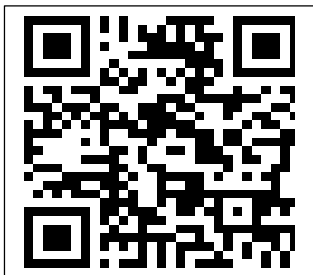
$$\text{From 1) } \alpha = \cos^{-1} \left( \frac{b^2 + c^2 - a^2}{2bc} \right)$$

$$\text{From 2) } \beta = \cos^{-1} \left( \frac{a^2 + c^2 - b^2}{2ac} \right)$$

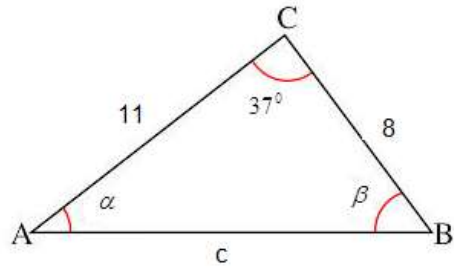
$$\text{From 3) } \gamma = \cos^{-1} \left( \frac{a^2 + b^2 - c^2}{2ab} \right)$$

Note: The law of cosines is used to solve triangles in the following cases:

- (i) Two sides and an included angle (SAS)
- (ii) Three sides (SSS)



**Example 1:** Solve the following triangle.



**(Round the answers up to 2 decimal places)**

**Solution:** Here

$$\gamma = 37^\circ, a = 8, \text{ and } b = 11$$

We are given two sides and an included angle, which is the SAS case.

**Step (1) Find the third side with the Cosine rule**

$$c^2 = a^2 + b^2 - 2ab \cos \gamma$$

$$c^2 = 8^2 + 11^2 - 2(8)(11) \cos 37$$

$$c = 6.6663\dots$$

**Step (2) Find any one of the remaining two angles with the Cosine rule**

For finding  $\beta$  the rule is

$$\beta = \cos^{-1} \left( \frac{a^2 + c^2 - b^2}{2ac} \right)$$

$$\beta = \cos^{-1} \left( \frac{8^2 + 6.6663^2 - 11^2}{2 \times 8 \times 6.6663} \right)$$

$$\beta = 96.7628 \dots$$

$$\beta = 96.76^\circ$$

(Rounded to 2 decimal places)

**Step (3) Find the third angle**

**Method 1)**  $\alpha + \beta + \gamma = 180^\circ$

$$\alpha + 96.76 + 37 = 180^\circ$$

$$\alpha = 46.24^\circ$$

$$\text{Method 2) } \alpha = \cos^{-1} \left( \frac{b^2 + c^2 - a^2}{2bc} \right)$$

$$\alpha = \cos^{-1} \left( \frac{11^2 + 6.6663^2 - 8^2}{2 \times 11 \times 6.6663} \right)$$

$$\alpha = 46.2371 \dots = 46.24$$

(Rounded to 2 decimal places)

$$\therefore \alpha = 46.24, \quad \beta = 96.76^\circ$$

and  $c = 6.67$  (Rounded to 2 decimal places)

### Alternative method after step 1

The angle can also be found with the help of sine rule but first will have to find the angle opposite the shorter of the two given sides. This angle will always be acute as in a triangle as there cannot be two obtuse angles in a triangle.

$$\frac{\sin \alpha}{8} = \frac{\sin \beta}{11} = \frac{\sin 37}{6.6663} \text{ ----- (1)}$$

Here we will have to find first  $\alpha$

$$\frac{\sin \alpha}{8} = \frac{\sin 37}{6.6663} \quad \text{or} \quad \sin \alpha = \frac{8 \sin 37}{6.6663}$$

$$\text{Hence, } \alpha = \sin^{-1} \left( \frac{8 \sin 37}{6.6663} \right)$$

$$\alpha = 46.2378 \dots^\circ = 46.24$$

(Rounded to 2 decimal places)

### **Step (3) Find the third angle**

$$\alpha + \beta + \gamma = 180^\circ$$

$$46.24 + \beta + 37 = 180^\circ$$

$$\beta = 96.76^\circ$$

$$\therefore \alpha = 46.24, \beta = 96.76^\circ$$

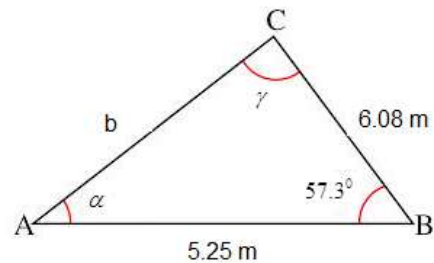
and  $c = 6.67$  (Rounded to 2 decimal places)

**Note:** If the third angle is found by Sine rule in this question then it will give wrong answer as it is an obtuse triangle.

### Class Activity 3

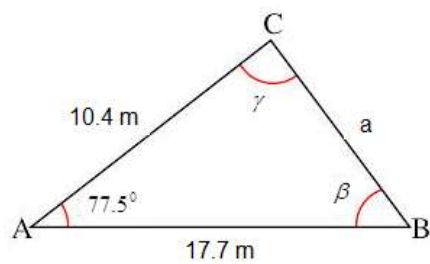
**(Round the answers up to 2 decimal places)**

1) Solve the following triangle:



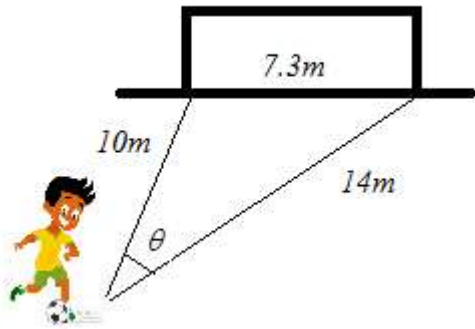


2) Solve the following triangle:



3) Find the missing side and angles of the given triangle:  $\alpha = 30^\circ$ ,  $b = 12$ ,  $c = 24$ .

4) A soccer player takes a shot on a standard net that is  $7.3\text{ m}$  wide. If the player is  $10\text{ m}$  from one goalpost and  $14\text{ m}$  from the other, through what angle  $\theta$  can a goal be made?



## WORKSHEET 2

### Section-A

Circle the correct answer in the following questions.

1) Sum of all three angles of a triangle is equal to .....

- (a)  $270^\circ$
- (b)  $90^\circ$
- (c)  $180^\circ$

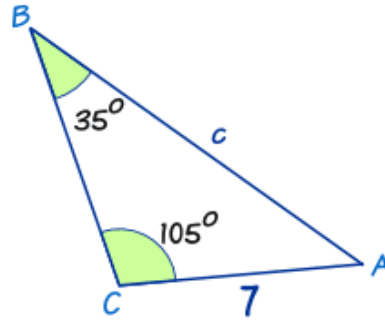
2) An oblique triangle has .....

- (a) one obtuse angle
- (b) a right angle
- (c) two obtuse angles

### Section-B

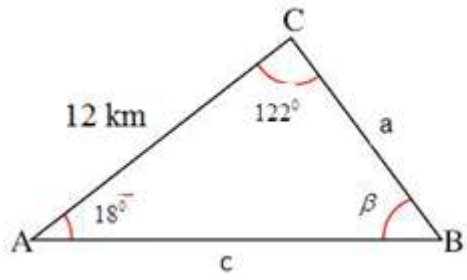
Show your solution step by step in the following questions. Round off the answer to two decimal places.

1) In the following triangle calculate side 'AB'.



**Solution:**

2) Solve the triangle:

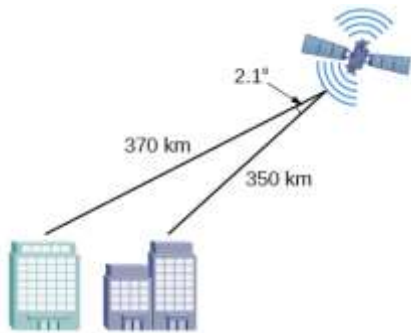


**Solution:**

3) Find the missing side and angles of the given triangle:  $\beta = 26^\circ$ ,  $a = 26$ ,  $c = 18$ .

**Solution:**

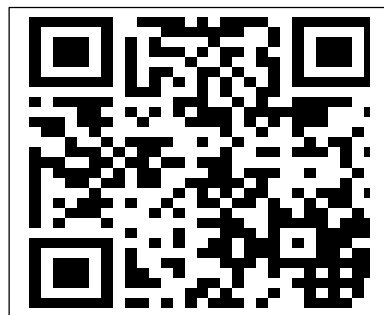
4) A satellite calculates the distances and angle shown in (Figure) (not to scale). Find the distance between the two cities. Round answers to the nearest tenth.



**Solution:**

5) A man leaves a point 'A' walking at 6.5 km/hr in a direction of  $70^\circ$ . A cyclist leaves the same point at the same time in a direction  $130^\circ$  travelling at a constant speed. If the walker and the cyclist are 80 km apart after 5 hours, find the average speed of the cyclist

**Solution:**



## REFERENCES AND INDICATIVE READING

<b>Title/Edition/Author</b>	<b>ISBN</b>
<b>College Algebra with Trigonometry-7<sup>th</sup> Edition</b> by K Raymond A., Ziegler Michael R., Byleen	ISBN-13: 978-0072368697 ISBN-10: 0072368691
<b>College Algebra and Trigonometry-5<sup>th</sup> Edition</b> by Margaret L. Lial, John Hornsby, David I. Schneider and Callie Daniels	ISBN-13: 978-0321671783 ISBN-10: 0321671783
<b>Bird's Basic Engineering Mathematics- 8<sup>th</sup> Edition</b> by John Bird	ISBN-13: 978-0367643706 ISBN-10: 0367643707
<b>Engineering Mathematics- 8<sup>th</sup> Edition</b> by K.A. Stroud and Dexter Booth	ISBN-13: 978-1352010275 ISBN-10: 1352010275
<b>Introduction to Statistics-3<sup>rd</sup> Edition</b> by Ronald E Walpole	ISBN-13 : 978-0024241405 ISBN-10 : 0024241407

### Websites/ HTML:

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<http://math.tutorvista.com>

[www.mathsisfun.com](http://www.mathsisfun.com)

[www.statcan.gc.ca](http://www.statcan.gc.ca)

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